1. What is SUPERSYMMETRY?

1.1 Introduction

Reminder: How do we decribe symmetries in particle physics?

U(1) - Gauge Symmetry : $\psi \rightarrow \psi'(x) = e^{iQ\alpha(x)}\psi(x)$

Q: charge operator
generator of symmetry

For electron: $Q\psi_e(x) = (-e)\psi_e(x)$

SU(2) - Gauge Symmetry:

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} \longrightarrow \begin{pmatrix} \psi_1'(x) \\ \psi_2'(x) \end{pmatrix} = e^{-i\theta_a(x)\frac{\tau^a}{2}} \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

τ^a : Pauli matrices , $\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\tau^\pm = \tau^1 \pm i\tau^2$, $\tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\tau^- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\tau^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\tau^- \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\mathcal{L}_\psi = \bar{\psi}(x)\, i\gamma^\mu\left(\partial_\mu - ig\frac{\vec{\tau}\cdot\vec{W}_\mu}{2}\right)\psi(x) - m\bar{\psi}(x)\psi(x)$$

$\rightsquigarrow \tau^\pm$ flips the isospin (and "has" isospin)

$\rightsquigarrow W^\pm$

In Supersymmetry: generator: Q, $\bar{Q}$

$$Q \,|\text{Boson}\rangle_{s=0} = |\text{FERMION}\rangle_{s=\frac{1}{2}}$$

$$\bar{Q} \,|\text{FERMION}\rangle_{s=\frac{1}{2}} = |\text{BOSON}\rangle_{s=0}$$

change spin by $\pm\frac{1}{2}$, Q: raises spin

$\bar{Q}$: lowers spin $(\bar{Q} = Q^*)$

Q: is a (spin-$\frac{1}{2}$) spinor (2-comp. Weyl spinor)

Want a Lagrangian: $\mathcal{L}_{SUSY}$

for which: $\int d^4x \; \delta \mathcal{L}_{SUSY} = \delta S_{SUSY} = 0$

invariant under SUSY transformations.

$\longrightarrow$ not possible within SM — need to <u>double</u> spectrum

EXAMPLE: ELECTRON

$(\psi_e)_L \, (s=\tfrac{1}{2}) \longleftrightarrow \varphi_{\tilde{e}_L} (s=0)$ Scalar Electron "selectron"

$(\psi_e)_R \, (s=\tfrac{1}{2}) \longleftrightarrow \varphi_{\tilde{e}_R} (s=0)$

$\tilde{e}_L$: "left-handed" selectron

$\tilde{e}_R$: "right-handed" selectron

These are scalars – no spin – and ~~thus~~ have no helicity. "L" & "R" are reminders of the SU(2) transformation properties

$SU(2) \updownarrow \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$ – doublet ; $\tilde{e}_R$ – singlet $(\tilde{\nu}_R)$

The superpartners $(e_L, \tilde{e}_L)$ have the same gauge properties :

$$[Q, \text{"gauge generators"}] = 0$$

$$[Q, T^a] = 0$$

EXTERNAL SYMMETRIES	INTERNAL SYMMETRIES
"Space-Time"	"Gauge Symmetries"
Ex: Lorentz group	SU(2)
$\boxed{\text{SUPERSYMMETRY}}$	

Coleman-Mandula / Haag-Łopuszanski-Sohnius Theorem:

"Supersymmetry is the only possible external symm. of the S-matrix, beyond Lorentz symmetry for which the S-matrix is not trivial."

$$S_{fi} = \langle \text{final} \,|\, \text{initial} \rangle = \mathbb{1} - i\, T_{fi}$$

End of the line for external symmetries

1.2. Supersymmetry Algebra

Reminder : $U(1)$: $[Q, Q] = 0$ only 1 generat.

algebra is trivial

$SU(2)$: $\left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = i\,\varepsilon_{ijk}\,\frac{\tau_k}{2}$

Supersymmetry is an external symmetry

→ transforms non-trivially under Lorentz trsf$^{\underline{ns}}$

Lorentz-Generators: P^μ — translations (Poincaré)

$J^{\mu\nu}$ — $\begin{cases} \text{rotations} : J^i = \frac{1}{2}\varepsilon^{ijk} J^{jk} \\ \text{boosts} : K^i = J^{0i} \end{cases}$

$[P^\mu, P^\nu] = 0$ translations commute

$[J^{\mu\nu}, P^\lambda] = i\left(g^{\nu\lambda}P^\mu - g^{\mu\lambda}P^\nu\right)$

$[J^{\mu\nu}, J^{\lambda\rho}] = i\left(g^{\nu\lambda}J^{\mu\rho} - g^{\mu\lambda}J^{\nu\rho} + g^{\mu\rho}J^{\nu\lambda} - g^{\nu\rho}J^{\mu\lambda}\right)$

Examples : $[J^{\mu\nu}, P^\lambda] = i(g^{\nu\lambda}P^\mu - g^{\mu\lambda}P^\nu)$

$(\mu\nu)=(ij)$, $\lambda=0$ $\quad [J^{ij}, P^0] = i(g^{j0}P^i - g^{i0}P^j) = 0$

rot$^{\text{ns}}$ commute w/ time translation

$(\mu\nu)=(12)$, $\lambda=3$ $\quad [J^{12}, P^3] = i(g^{23}P^1 - g^{13}P^2) = 0$

rot$^{\text{n}}$ around 3-axis commutes w/ transl$^{\text{n}}$ along 3-axis.

$(\mu\nu)=(03)$, $\lambda=0$ $\quad [J^{03}, P^0] = i(g^{30}P^0 - g^{00}P^3) = -iP^3$

boost along 3-axis does not commute with time translation

Now what happens when we include spinor Q ?

First make 4-comp. Majorana spinor

$$Q_M = \begin{pmatrix} Q_\alpha \\ \bar{Q}^{\dot{\alpha}} \end{pmatrix}$$

a) $[Q_\mu , P^\mu] = 0$

spin is unaffected by translations.

b) $[Q_\mu , J^{\mu\nu}] = \frac{1}{2}\sigma^{\mu\nu} Q_\mu$, $\sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu , \gamma^\nu]$

c) $\{Q_\mu , \bar{Q}_\mu\} = 2\gamma^\mu P_\mu$, d) $\{Q_\mu , Q_\mu\} = 0$

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b) spin is affected by rot$^{\text{ns}}$ — Thomas precession

c) $\{Q_\mu , \bar{Q}_\mu\} \rightarrow$ space-time translation

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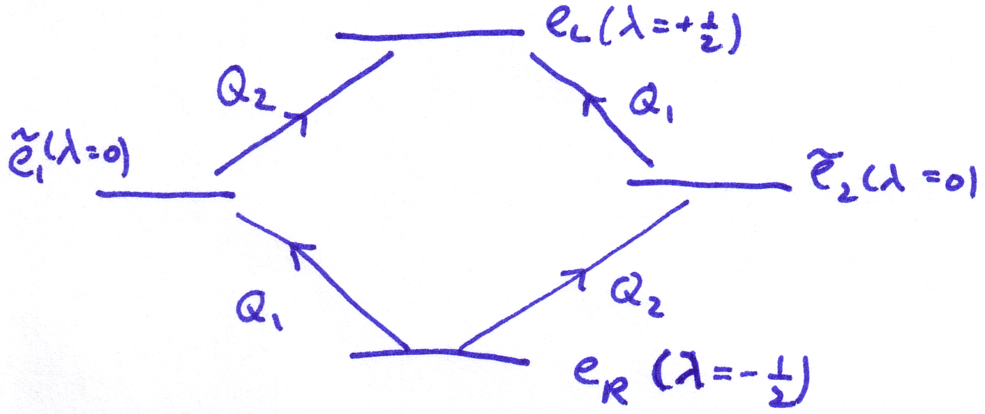
Q: raises by spin $\frac{1}{2}$, $\bar{Q}$: lowers by spin $\frac{1}{2}$

$$\tilde{e}_L (s=0) \underset{\bar{Q}}{\overset{Q}{\rightleftarrows}} e_L (s=\tfrac{1}{2})$$

In principle could have more than one SUSY Generator: Q_1, Q_2 $(\bar{Q}_1, \bar{Q}_2)$



$$\{Q_1, Q_2\} = 0$$

Since $[Q_{1,2}, T^a] = 0$ $\Rightarrow$ e_L & e_R would have same gauge quantum numbers. $\lightning$

This is called $N=2$ supersymmetry and it is phenomenologically excluded.

$$[Q, P^\mu] = 0$$

$$\Rightarrow [Q, P^2] = P_\mu [Q, P^\mu] + [Q, P_\mu] P^\mu = 0$$

$$\Rightarrow [Q, (\text{Mass})^2] = 0$$

$$\Rightarrow \quad \text{Mass}(e^-) = \text{Mass}(\tilde{e}^-)$$

selectron

$\Rightarrow$ supersymmetry predicts a

$(\text{spin} = 0, \; Q = -1, \; m = 511\,\text{keV})$ - particle

- EXPERIMENTALLY EXCLUDED.

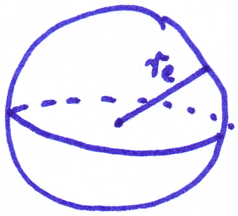
$\Rightarrow$ supersymmetry must be broken

$$\text{Mass}(\tilde{e}^-) \gg \text{Mass}(e^-)$$

how much bigger?

1.3. HIERARCHY PROBLEM

Consider first electron as a charged sphere

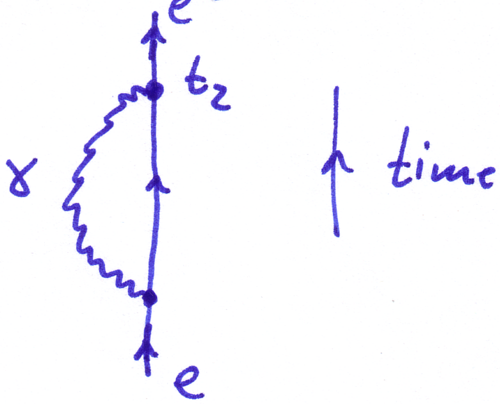


Using Maxwell's Eq^{ns} compute Coulomb self-energy

$$E_{self} = \frac{3}{5} \frac{e^2}{r_e^2}$$

- This is linearly divergent for $r_e \to 0$, $E_{self} \nearrow \infty$
- This is the problem of a "point particle" in classical E & M
- In Quantum Mechanics compute the diagram

  - single electron theory
  - use time ordered perturbation theory



$$\Rightarrow \quad m_e c^2 = m_e^\circ c^2 + E_{self}$$

as above

$r_e \searrow 0 \Rightarrow$ lower m_e° to keep $m_e c^2 = 511$ keV

But at $r_e = 4$ fm $\Rightarrow E_{self} \geq 511$ keV

$\Rightarrow m_e^0 < 0$?

At LEP (CERN) [Large Electron Positron Collider

$E_{CM} = 90 - 200$ GeV]

point-like nature of electron is confirmed

for $\boxed{r_e \leq 10^{-3} \text{ fm}} \ll 4$ fm

What to do?

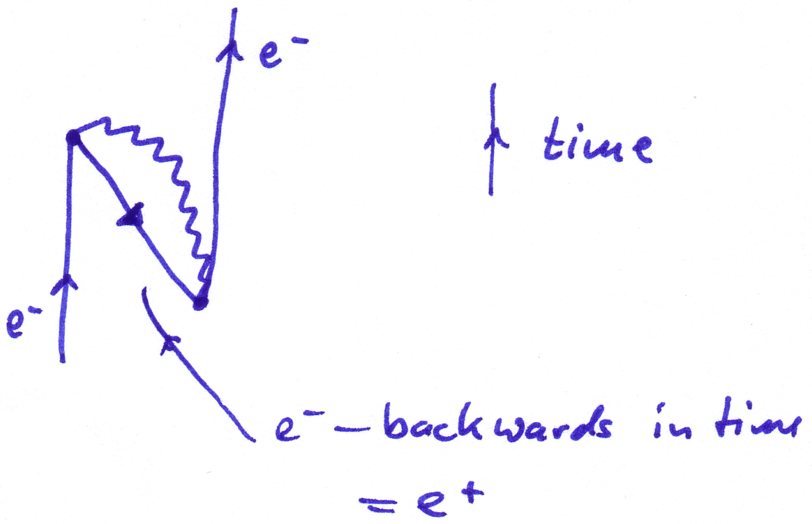
Introduce a new particle: e^+ – Positron

$m(e^+) = m(e^-)$

$Q(e^+) = -Q(e^-)$

Same quantum numbers – except charge, which is opposite

New diagram:



$$\Rightarrow \quad E_{self} = \frac{3}{4\pi} \frac{e^2}{r_{compt}} \ln\left(\frac{m_e c \cdot r_e}{\hbar}\right)$$

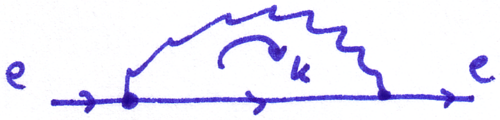
only log divergence

Historically this was first done by Viktor Weisskopf, 1934 — he actually got it wrong first, but was helped out by Furry.

If I set $r_e = l_{Planck}$ (the smallest length scale in physics)

$$\Rightarrow \quad E_{self} = \mathcal{O}(0.1)\, m_e c^2 \quad \text{— a mild correction}$$

In QED just one diagram:



$$\Delta m_e = \frac{\alpha}{\pi} m_e \ln\left(\frac{\Lambda^2 + m_e}{m_e^2}\right), \quad k < \Lambda$$

$$\Lambda = M_{Pl} \Rightarrow \Delta m_e = \frac{1}{4} m_e$$

What have we done?

We have doubled particle content

$\longrightarrow$ reduced divergence

We have also extended the symmetry.

We now have an extra CPT – invariance.

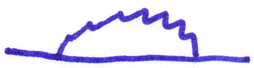
In the SM we have particles

$S = 1$ — Gauge bosons

$S = \frac{1}{2}$ — Quarks & Leptons

$S = 0$ — Higgs boson

$S = 1$  correction to photon mass $= 0$, protected by gauge symm.

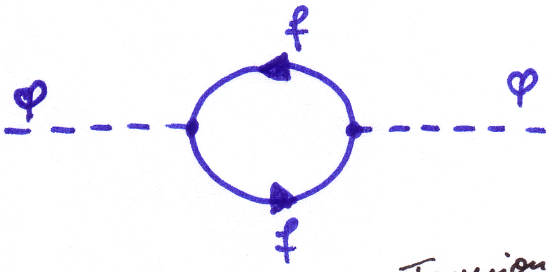
$S = \frac{1}{2}$  $\sim m_e \cdot \ln\left(\frac{\Lambda}{m_e}\right)$

mild log. corr.

vanishes when $m_e \to 0$

chiral symm.

Scalar Self-Energy: (see M. Drees hep-ph/9611409)



Fermion loop

$$\Pi^{f}_{\varphi\varphi}(q^2=0) = -N(f)\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\left(\frac{i\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\left(\frac{i\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\right]$$

$$= -N(f)\,\frac{\lambda_f^2}{2}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{(\not{k}+m_f)(\not{k}+m_f)}{(k^2-m_f^2)^2}\right]$$

$$= -2N(f)\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\,\frac{k^2+m_f^2}{(k^2-m_f^2)^2}$$

$$= -2N(f)\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\left[\underbrace{\frac{1}{k^2-m_f^2}}_{1}+\underbrace{\frac{2m_f^2}{(k^2-m_f^2)^2}}_{2}\right]$$

Recall: $d^4k = d\Omega_4\,dk\,k^3$ $\quad\left(\int d\Omega_4 = 2\pi^2\right)$

1 $\displaystyle\int\int_0^{\Lambda}\frac{d\Omega_4\,dk\,k^3}{(2\pi)^4}\,\frac{1}{k^2-m_f^2} \sim \frac{1}{8\pi^2}\Lambda^2$

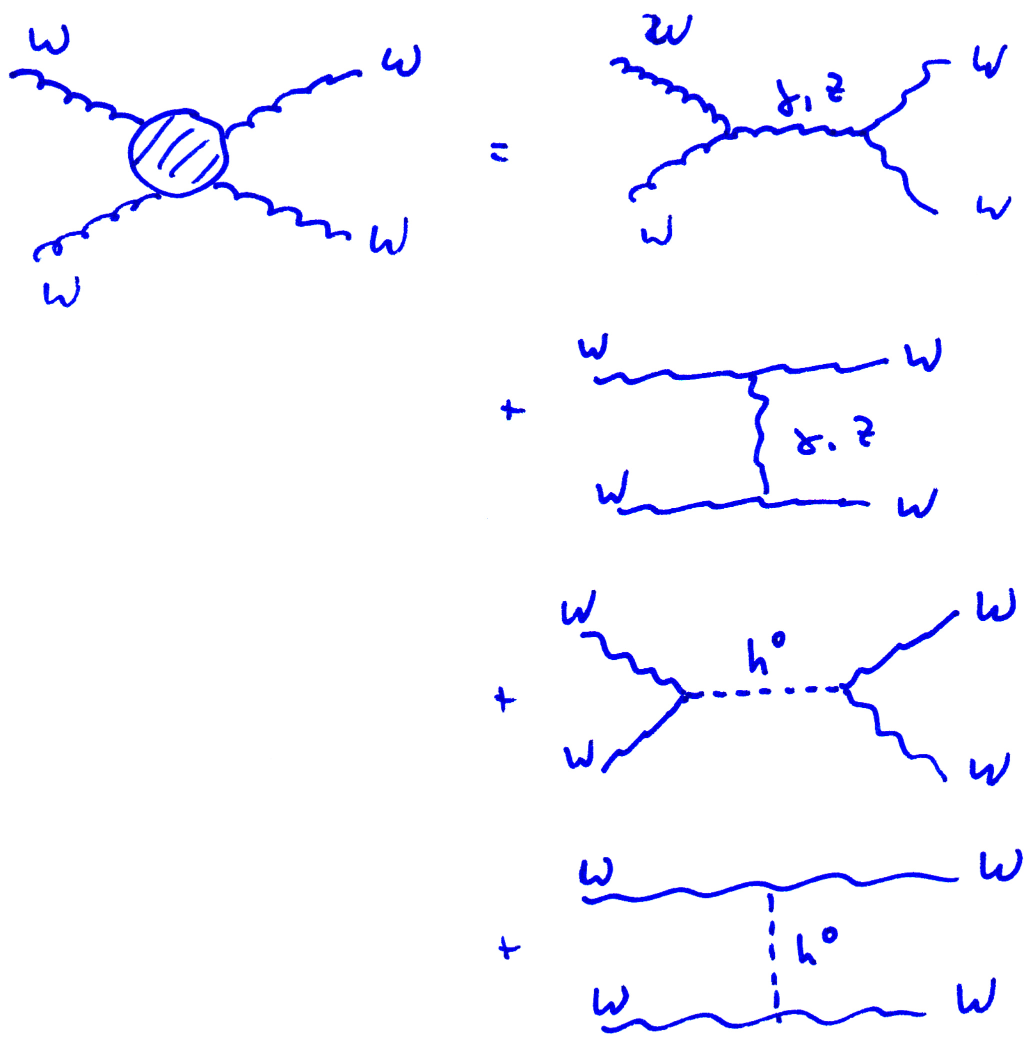
2 $\displaystyle\int\frac{d\Omega^4}{(2\pi)^4}\int_0^{\Lambda}dk\,k^3\,\frac{2m_f^2}{(k^2-m_f^2)^2} \sim \ln\frac{\Lambda^2}{m_f^2}$

φ is not protected by a symmetry.

This is a disaster in SM, need

$$M_{Higgs} \lesssim \mathcal{O}(1\,\mathrm{TeV})$$

by unitarity



violates unitarity if $M_h \gtrsim 1\,\mathrm{TeV}$

Introducing Λ only makes sense if there is a new scale: $\Lambda \sim M_{GUT}\,(10^{16}\,\text{GeV})$

$M_{see\text{-}saw}\,(10^{10}\,\text{GeV})$

$M_{Planck}\,(10^{19}\,\text{GeV}) \longleftarrow$

If there is **no** new scale, then can absorb **the** Λ^2-divergence into $m_{h^0}^2$ – renormalise – just like the log divergence.

But we are confident, that new physics must arise **at** the latest **at** M_{Planck}.

And even if we can absorb the ∞ when including the new physics — we have the finite correction:

$$\Delta m_h^2 = \frac{N(f)\,\lambda_f^2}{4\pi^2}\, m_f^2 \left(\frac{\Lambda^2}{\Lambda^2 + m_f^2}\right)$$

If there are any new particles at scale Λ coupling to h we are doomed again.

One could get rid of correction via a counter term in $\mathcal{L}$

This counter term: $-b \cdot \Lambda^2$ would have to be fine-

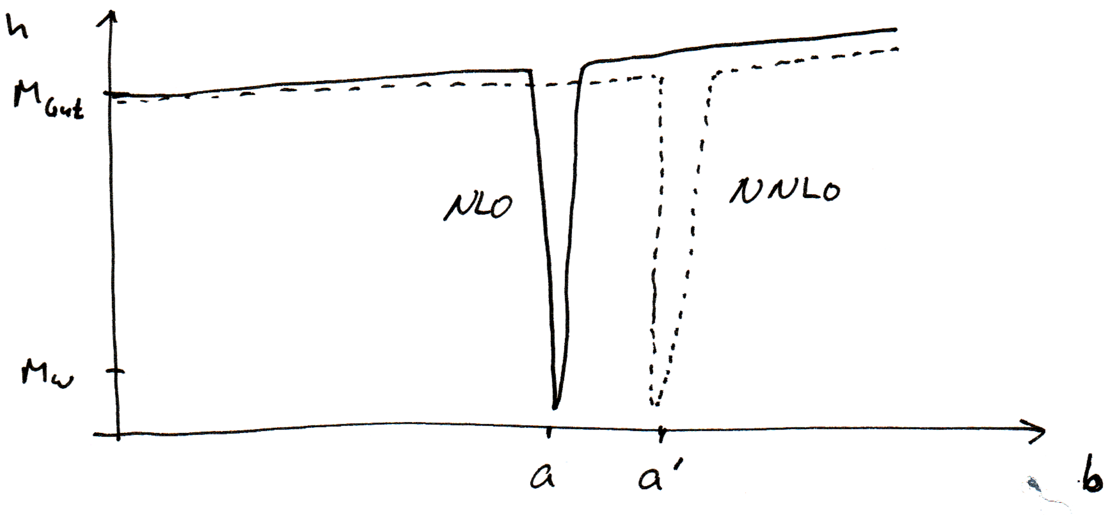
-tuned to a fantastic precision

$$(m_h)^2 = (m_h^0)^2 + \underbrace{a \Lambda^2}_{\substack{\text{rad.} \\ \text{corr.}}} - b \Lambda^2$$

$$\Rightarrow \quad (a-b) \cdot \Lambda^2 = \mathcal{O}((1\ \text{TeV})^2)$$

$$\Rightarrow \quad a-b = \mathcal{O}(10^{-29}) \quad !$$

This is a huge fine-tuning



Different fine-tuning in each order of pert. theory

Prediction ?

Standard Modell has a deep problem

$\longrightarrow$ need new physics

First, let us ask if there is a higher scale.

* Surprisingly, this question is experimentally accessible.

Plot $\longrightarrow$

For $130\,\mathrm{GeV} \lesssim m_h \lesssim 200\,\mathrm{GeV}$ avoid these problems.

Hambye & Riesselmann
hep-ph/9708416

But if we find $m_h = 400\,\mathrm{GeV}$, for example

$$\Rightarrow \Lambda \lesssim 100\,\mathrm{TeV}$$

Further hints of a higher scale

- Gravity — G_N: Newton's constant

$$G_N =$$

$$\Rightarrow \quad r = l_{Pl} = \frac{1}{M_{Pl}} \qquad \text{Gravity} \sim \text{order } 1$$

- coupling constant unification

Measure

$$\left.\begin{aligned} \alpha^{-1}_{QED}(M_Z) &= 127.922 \pm 0.027 \\ \sin^2\theta_W(M_Z) &= 0.2228 \pm 0.0004 \end{aligned}\right\} \quad \begin{aligned} &\rightarrow \alpha_1 \\ &\rightarrow \alpha_2 \end{aligned}$$

$$\alpha_s(M_Z) = 0.1172 \pm 0.0020$$

$\xrightarrow{\text{Plot}}$

- Neutrino Masses : $m_{\nu_i} \ll m_{l^\pm_i}$ (more later)

see-saw: $m_{\nu_i} = \mathcal{O}\left(\frac{m^2_{u_i}}{M}\right)$, $M = \mathcal{O}(10^{10}\,\text{GeV})$

- Baryon asymmetry

$$\frac{N_B - N_{\bar{B}}}{N_\gamma} \simeq 10^{-10}$$

Not possible in SM

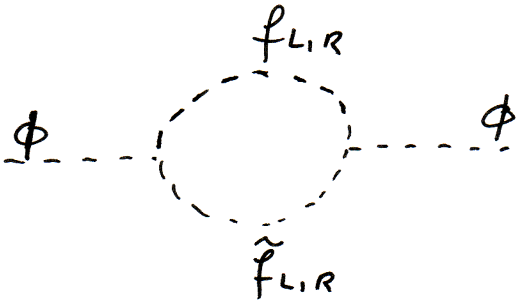
Supersymmetric Solution to the hierarchy problem

RECALL: WE DOUBLED THE SPECTRUM

$$(e_L^-, e_R^-) \oplus (\tilde{e}_L^-, \tilde{e}_R^-)$$

$\Rightarrow$ extra diagrams

$$\mathcal{L}_{\phi\tilde{f}} = -\frac{\tilde{\lambda}_f^2}{2}\Phi^2\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right) - v\tilde{\lambda}_f^2\,\phi\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right)$$



Scalar (boson) loops $\Rightarrow$ opposite sign

$$\Pi_{\phi\phi}^{\tilde{f}}(0) = \tilde{\lambda}_f^2\, N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2}\right] \qquad (*)$$

$$+ (\tilde{\lambda}_f^2 v)^2 N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2 - m_{\tilde{f}_L}^2)^2} + \frac{1}{(k^2 - m_{\tilde{f}_R}^2)^2}\right]$$

(*) Exactly cancels Λ^2-divergence, if

$$\boxed{\tilde{\lambda}_f = \lambda_f \quad , \quad N(\tilde{f}_L) = N(\tilde{f}_R) = N(f)}$$

Symmetry

These are just the conditions for supersymmetry!

Same quantum numbers $\Rightarrow$ same couplings

Same degrees of freedom $\Rightarrow$ $N(\tilde{f}) = N(f)$

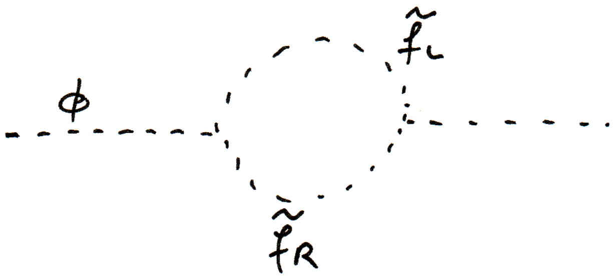
<u>Note</u>: Cancellation of Λ^2-diverg. independent of $m^2_{\tilde{f}_{L,R}}$

<u>SUSY - BREAKING</u>: (we will come back to this.)

$$\Delta \mathcal{L} = \Delta m^2_{\tilde{f}_L} |\tilde{f}_L|^2 + \Delta m^2_{\tilde{f}_R} |\tilde{f}_R|^2 + \left(\frac{\lambda_f}{\sqrt{2}} A_f \, \phi \, \tilde{f}_L \tilde{f}_R + \text{h.c.}\right)$$

A_f : tri-linear susy breaking parameter

$\dim[A_f] = \text{GeV}$ (energy)



$$\Delta \Pi^{\tilde{f}}_{\phi\phi}(0) = (\lambda_f A_f)^2 N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2_{\tilde{f}_L})(k^2 - m^2_{\tilde{f}_R})}$$

also no Λ^2- divergence.

Final answer (for $m_{\tilde{f}_L} = m_{\tilde{f}_R} = m_{\tilde{f}}$)

$$\Pi_{\phi\phi}^{f+\tilde{f}} = i \, \frac{\lambda_f^2 N(f)}{16\pi^2} \Big[-2m_f^2 \left(1 - \log \frac{m_f^2}{\mu^2}\right) + 4m_f^2 \log \frac{m_f^2}{\mu^2}$$

$$+ 2m_{\tilde{f}}^2 \left(1 - \log \frac{m_{\tilde{f}}^2}{\mu^2}\right) - 4m_f^2 \log \frac{m_{\tilde{f}}^2}{\mu^2} - |A_f|^2 \log \frac{m_{\tilde{f}}^2}{\mu^2} \Big]$$

$$= 0 \quad , \quad \text{for} \quad \left.\begin{matrix} A_f = 0 \\ m_{\tilde{f}} = m_f \end{matrix}\right\} \text{SUSY limit}$$

Note: $m_{\tilde{f}}^2$ - term above

$\Rightarrow$ if SUSY is broken only have

$m_\phi^2 \leq (1 \text{ TeV})^2$ if $m_{\tilde{f}}^2 \leq (1 \text{ TeV})^2 \cdot \left(\frac{16\pi^2}{\lambda^2 N}\right)$

$\Rightarrow$ SUSY must be broke at $\mathcal{O}(1 \text{ TeV})$

to solve hierarchy problem

$\boxed{\text{This is the motivation for low-energy SUSY}}$