3. MSSM – Minimal Supersymmetric Standard Model

3.1. PARTICLE CONTENT

RECALL STANDARD MODEL:

- 1 FAMILY OF PARTICLES, SPIN $= \frac{1}{2}$ $(\times 3)$

$$\underbrace{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , e_R}_{\text{LEPTONS}} \; ; \; \underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L , u_R , d_R}_{\text{QUARKS}}$$

- 1 HIGGS DOUBLET, SPIN=0

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$$

- 12 GAUGE BOSONS: (SPIN = 1)

$$\underbrace{B}_{U(1)_Y} , \underbrace{W_3^0 , W^\pm}_{SU(2)} \qquad \underbrace{g_{i=1,\dots,8}}_{SU(3)}$$

$(\gamma, Z^0, W^\pm)$

SUPERSYMMETRY $\longrightarrow$ MUST DOUBLE SPECTRUM + X

SUPERSYMMETRY PARTICLES

- $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , e_R^c$ $\qquad$ $\tilde{L}_e = \begin{pmatrix} \tilde{\nu}_{eL} \\ \tilde{e}_L \end{pmatrix}, \tilde{e}_R^c$

$$\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L , u_R^c , d_R^c}_{\psi_L} \qquad \underbrace{\tilde{Q}_1 = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \tilde{u}_R^c , \tilde{d}_R^c}_{\varphi}$$

supermultiplet

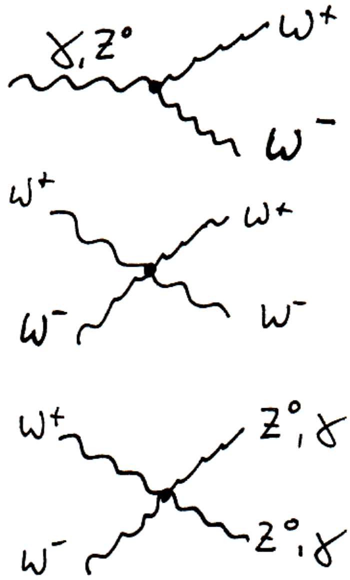
- $$\underbrace{H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} ; \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}}_{\varphi} \qquad \underbrace{\begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix} \quad \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix}}_{\psi_L}$$

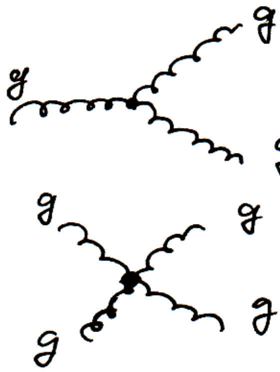
- $$\underbrace{\begin{matrix} B, \; W_3^0 \; (\gamma, Z^0) \\ W^\pm \\ g_{i=1,\dots,8} \end{matrix}}_{A_\mu^a} \qquad \underbrace{\begin{matrix} \tilde{B}, \; \tilde{W}_3^0 \; (\tilde{\gamma}, \tilde{Z}^0) \\ \tilde{W}^\pm \\ \tilde{g}_{i=1,\dots,8} \end{matrix}}_{\lambda^a}$$

3.2 MSSM – Lagrangian & Feynman Diagrams

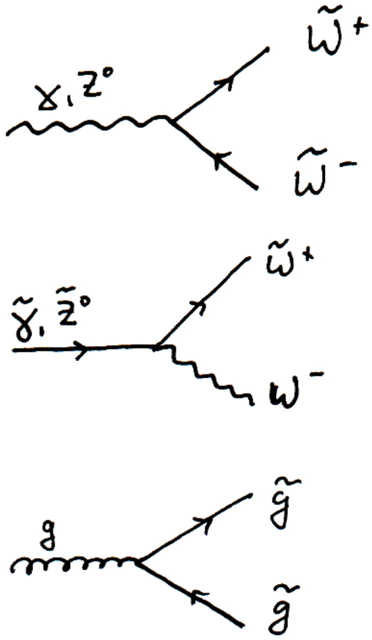
3.2.1 Gauge Interactions

a) $F^a_{\mu\nu} F^{\mu\nu a}$



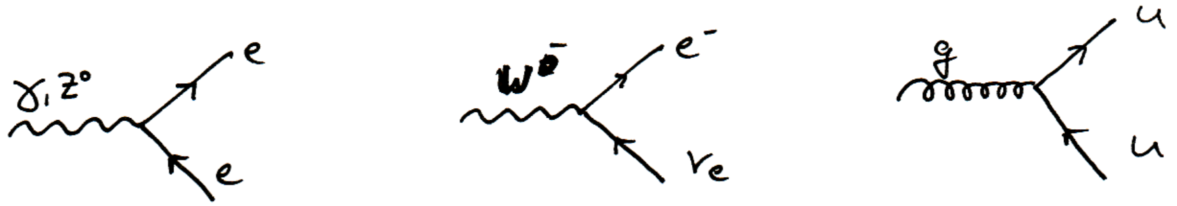


b) $\bar{\lambda}^a \gamma^\mu D_\mu \lambda^a$



Note : Take 2 SUSY particles $\rightarrow$ non SUSY $\Rightarrow$ SM-vertex
$(\tilde{g})$ (g)

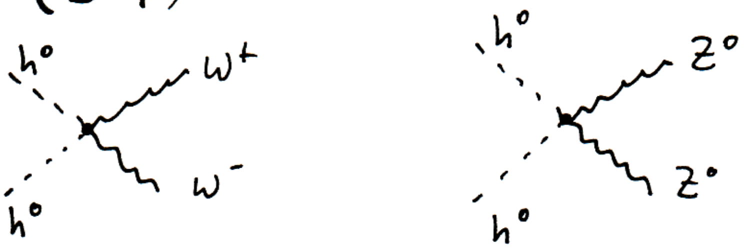
c) $\bar{\psi}_{Li}\, \gamma^\mu D_\mu \psi_{Lj}$ — SM



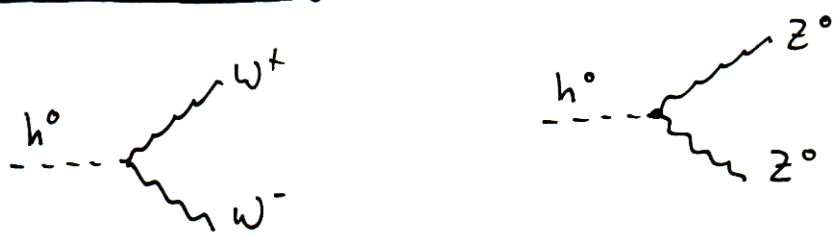
→ whoops

d) $(D_\mu \phi)^\dagger (D^\mu \phi)$

SM:

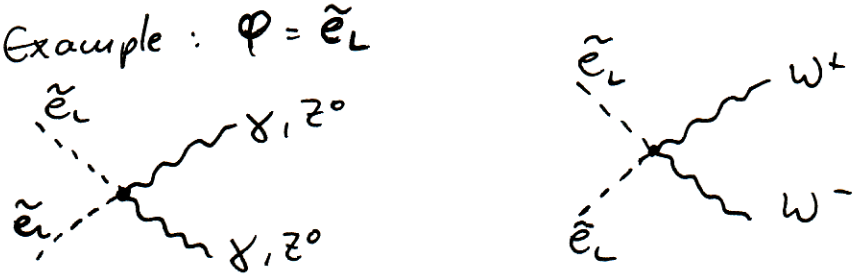


After SU(2)×U(1) breaking:



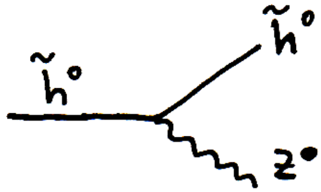
$\to M_W$ $\quad\quad$ $\to M_Z$

SUSY: Example : $\varphi = \tilde{e}_L$

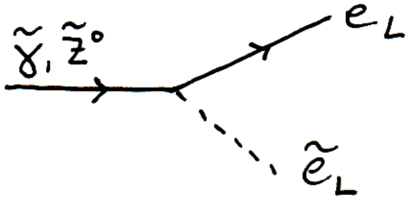


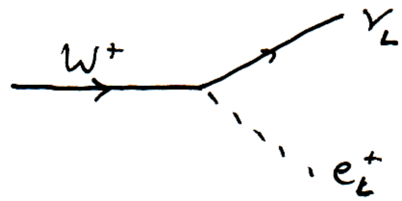
↑ does not exist for $\tilde{e}_R$

c) $\overline{\Psi}_{Li} \gamma^{\mu} D_{\mu} \Psi_{Lj}$ – SUSY

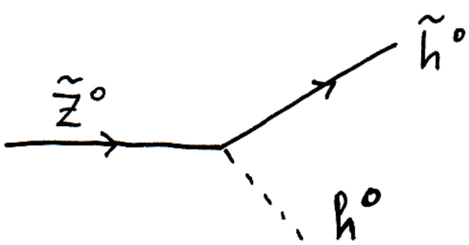


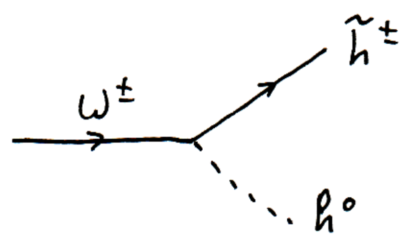
e) $(\bar{\lambda}^a \psi_{L\,i})\, T^a_{ij}\, \varphi_j$



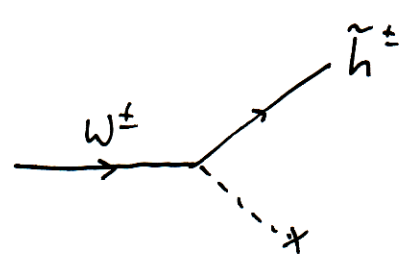


Note again: change 2 SUSY $(\tilde{e}_L, \tilde{\gamma}) \to$ SM (e_L, γ) get SM vertex





AFTER $SU(2) \times U(1)$ BREAKING



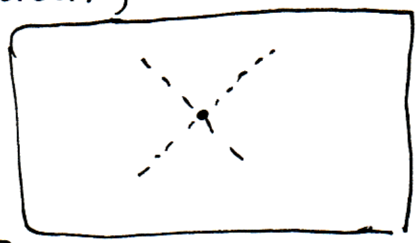
What is this?

f) $\frac{1}{2} \sum_a D^a D^a$; $D^a = g\left(\varphi_i^* T_{ij} \varphi_j\right)$

Example: $U(1)_Y$, $T_{ij} = Y \cdot \delta_{ij}$

(Summarize Y-charges in a moment;

recall: $Q = T_3 + \frac{1}{2} Y$



• $2 g_Y^2 \Big[\frac{1}{2}|H_2^0|^2 + \frac{1}{2}|H_2^+|^2 - \frac{1}{2}|H_1^0|^2 - \frac{1}{2}|H_1^-|^2 - \frac{1}{2}|\tilde{e}_L|^2 - \frac{1}{2}|\tilde{\nu}_L|^2$

$+ |\tilde{e}_R^c|^2 + 3 \cdot \frac{1}{6}\left(|\tilde{u}_L|^2 + |\tilde{d}_L|^2\right) - 3 \cdot \frac{2}{3}|\tilde{u}_R^c|^2 + 3 \cdot \frac{1}{3}|\tilde{d}_R^c|^2 \Big]^2$

$\Rightarrow$ quartic Higgs interaction is a gauge coupling

(further term from $SU(2)_L$)

but this is only quartic coupling

$\Rightarrow$ $M_{h^0} < M_Z$ at tree-level (more later)

• What happens when $SU(2) \times U(1)_Y \longrightarrow U(1)_{EM}$?

$\rightarrow$ extra mass terms for scalars proportional to

$g^2 \cdot \langle vev \rangle^2 = \begin{cases} M_W^2 \\ M_Z^2 \end{cases}$

3.2. Superpotential Interactions

What is the MSSM superpotential?

Recall: $W = \frac{1}{2} M^{ij} \varphi_i \varphi_j + \frac{1}{6} y^{ijk} \varphi_i \varphi_j \varphi_k$

$$\mathcal{L}_W = \frac{1}{2} \frac{\delta^2 W}{\delta\varphi_i \delta\varphi_j} \overline{\psi^c_{Li}} \psi_{Lj} + \left| \frac{\delta W}{\delta \varphi_i} \right|^2$$

$\Rightarrow$ if we know "W", $\mathcal{L}$ and thus interactions are determined.

- Note $\mathcal{L}_W \subset \mathcal{L}_{TOT}$. Since $\mathcal{L}_{TOT}$ MUST BE GAUGE INVARIANT, SO MUST $\mathcal{L}_W$.

- What are the gauge properties of all φ_i?

GAUGE – REPRESENTATIONS

φ_i	$SU(3)$	$SU(2)$	$\frac{1}{2}U(1)_Y$
$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$\tilde{e}_R^c$	1	1	$+1$
$\tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$\tilde{u}_R^c$	3	1	$-\frac{2}{3}$
$\tilde{d}_R^c$	3	1	$+\frac{1}{3}$
$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$	1	2	$-\frac{1}{2}$
$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$	1	2	$+\frac{1}{2}$

$W = ?$

EXAMPLE: $\tilde{L} H_1 \tilde{e}_R^c$ $\qquad$ $\tilde{Q} H_1 \tilde{d}_R^c$ $\qquad$ $\tilde{Q} H_2 \tilde{u}_R^c$

$\frac{1}{2}Y$: $-\frac{1}{2} - \frac{1}{2} + 1 = 0$ $\qquad$ $\frac{1}{6} - \frac{1}{2} + \frac{1}{3} = 0$ $\qquad$ $\frac{1}{6} + \frac{1}{2} - \frac{2}{3} = 0$

These are just the terms which give mass to leptons and quarks

TOTAL POSSIBLE W, GIVEN FIELD CONTENT & $SU(3)_C \times SU(2)_L \times U(1)_Y$ – SYMMETRY:

$$W = (h^E)_{ij}\, \tilde{L}_i H_1 \tilde{e}^c_{Rj} + (h^D)_{ij}\, \tilde{Q}_i H_1 \tilde{d}^c_{Rj} + (h^U)_{ij}\, \tilde{Q}_i H_2 \tilde{u}^c_{Rj}$$

$$+ \mu H_1 H_2$$

$$+ \lambda_{ijk} \tilde{L}_i \tilde{L}_j \tilde{e}^c_{Rk} + \lambda'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{d}^c_{Rk} + \lambda''_{ijk} \tilde{u}^c_{Ri} \tilde{d}^c_{Rj} \tilde{d}^c_{Rk}$$

$$+ \kappa_i \tilde{L}_i H_2$$

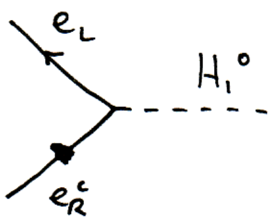
NOTE: NEEDED $H_2 \longrightarrow$ up-quark mass

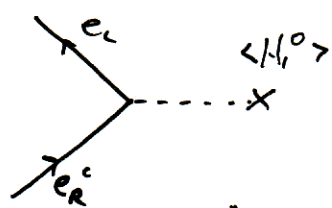
What are the Lagrangian terms?

(a) $\dfrac{\delta^2 W}{\delta\varphi_i \delta\varphi_j} \bar{\psi}_i \psi_j$

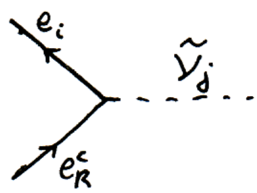
EXAMPLE: $\dfrac{\delta^2 W}{\delta \tilde{L}_i\, \delta \tilde{e}^c_{Rj}} \bar{L}_i e^c_{Rj} = (h^E)_{ij} (\bar{L}_i H_1) e^c_{Rj} + \lambda_{ijk} \bar{L}_i \tilde{L}_j e^c_{Rj}$

Feynman Graphs:





$\uparrow \Rightarrow m_e$

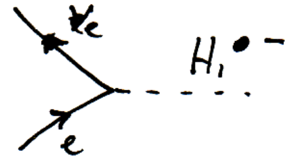


$\leftarrow$ violates lepton number

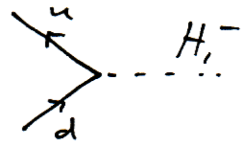
Further Terms

$\lfloor W \rfloor$

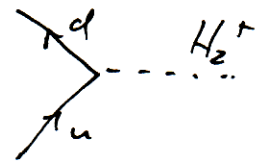
$\tilde{L}_i H_1 \tilde{e}^c_{Rj} \longrightarrow m_e$,



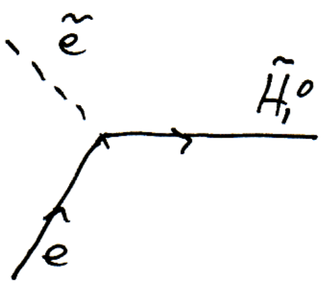
$\tilde{Q}_i H_1 \tilde{d}^c_{Rj} \longrightarrow m_d$,



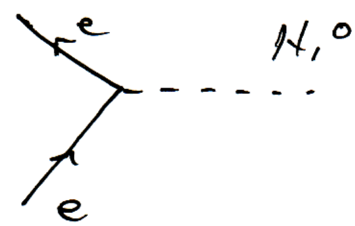
$\tilde{Q}_i H_2 \tilde{u}^c_{Rj} \longrightarrow m_u$



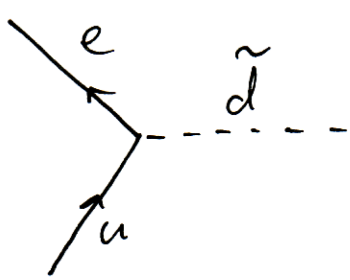
AGAIN, SWAP 2 SUSY → SM PARTICLES, GET SM-VERTEX



$\longrightarrow$



$\tilde{L}_i \tilde{Q}_j \tilde{D}^c_k$



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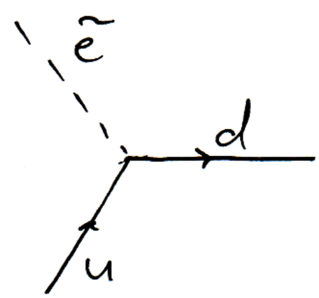

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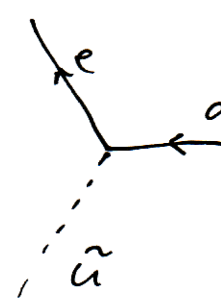


LEPTON NUMBER VIOLATION → COME BACK TO **THIS**

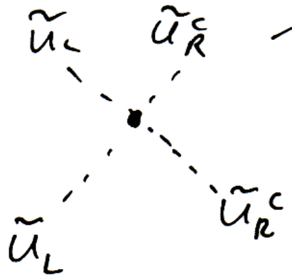
$$\frac{\delta^2 W}{\delta H_1 \delta H_2} \overline{\Psi}_{H_1} \Psi_{H_2} = \mu \overline{\Psi}_{H_1} \Psi_{H_2}$$

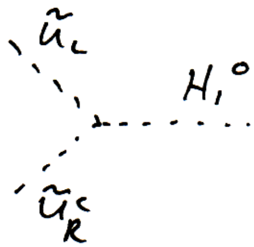
HIGGS MASS MIXING TERM

(b) $\left| \frac{\delta W}{\delta \varphi_i} \right|^2$

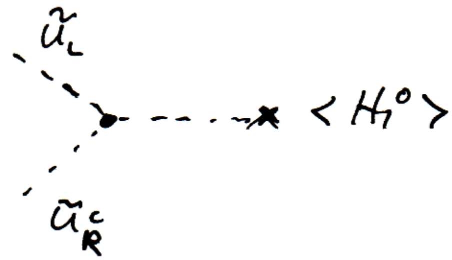
EXAMPLE : $\left| \frac{\delta W}{\delta H_2} \right|^2 = \left| \tilde{Q}_i \tilde{u}^c_{R_j} + \mu H_1 \right|^2$

$$= \left| \tilde{Q}_i \tilde{u}^c_{R_j} \right|^2 + \mu^2 |H_1|^2 + 2\mu \, \mathrm{Re} \left(H_1 \tilde{Q}_i \tilde{u}^c_{R_j} \right)$$





$\downarrow$ SU(2)xU(1) breaking

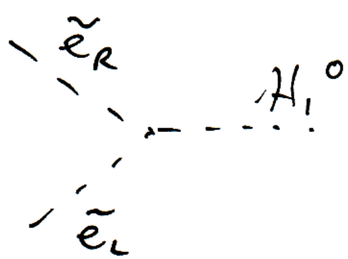


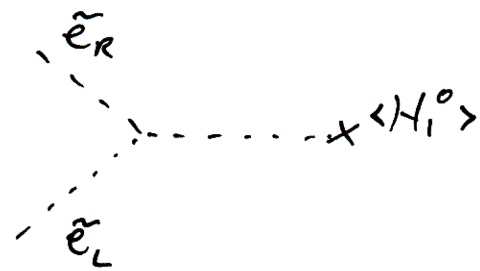
$\tilde{u}_L \tilde{u}^c_R$ - MIXING

3.3. SUSY BREAKING

$A_{ijk}\,\varphi_i\varphi_j\varphi_k$

EXAMPLE: $A_{ij}\,\tilde{L}_i H_1 \tilde{e}^c_{Rj}$





$(\tilde{e}_R - \tilde{e}_L)$ – MIXING

AFTER $SU(2)\times U(1)$ BREAK.

MAIN EFFECT

3.4. SUSY MASS MIXING

(1) SLEPTONS (SCALAR LEPTONS) $(\tilde{e}, \tilde{\mu}, \tilde{\tau})$

$(\text{Mass matrix})^2$: $(\tilde{e}_L, \tilde{e}_R)\, M^2_{\tilde{\ell}} \begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix}$

$$M^2_{\tilde{\ell}} = \begin{pmatrix} m_\ell^2 + m^2_{\tilde{\ell}_L} - \left(\frac{1}{2} - \frac{1}{3} s_W^2\right)\cos(2\beta) M_Z & -m_\ell (A_\ell + \mu \tan\beta) \\ -m_\ell (A_\ell + \mu \tan\beta) & m_\ell^2 + m^2_{\tilde{\ell}_R} - \frac{1}{3} s_W^2 \cos(2\beta) M_Z^2 \end{pmatrix}$$

Origin of terms:

m_ℓ^2 : $\left|\frac{\delta W}{\delta \tilde{e}_R}\right|^2$, $\left|\frac{\delta W}{\delta \tilde{L}}\right|^2$

$m^2_{\tilde{\ell}_L}$: SOFT BREAKING TERM $\tilde{m}^2 \varphi_i^+ \varphi_i$

$\sim \cos(2\beta) M_Z^2$: $(D^a D^a)$

$m_\ell A_\ell$: SOFT BREAKING : $A_{ij} \cdot y_{ij} \cdot (\tilde{L}_i \tilde{e}^c_{R_j}) \langle H_1^0 \rangle$

$m_\ell \mu \tan\beta$: $\left|\frac{\delta W}{\delta H_1}\right|^2$

$$\tan\beta = \frac{\langle H_2^0 \rangle}{\langle H_1^0 \rangle}$$

(2) SQUARKS (ANALOGOUS)

(3) CHARGINOS $\tilde{\chi}^{\pm}_{i=1,2}$ ($\tilde{C}^{\pm}_{1,2}$, new notation)

$(\tilde{W}^{\pm}, H^{\pm}_{1,2})$ — HAVE SAME QUANTUM NUMBERS AFTER

$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$

SPIN $= \frac{1}{2}$, CHARGE $= \pm 1$

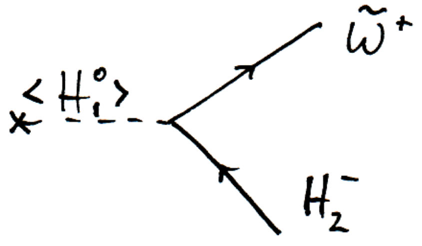
NO - COLOUR

$\Rightarrow$ THEY CAN MIX

$$(\tilde{W}^{+}, H_2^{+}) \, (M_{\pm}) \begin{pmatrix} \tilde{W}^{-} \\ H_1^{-} \end{pmatrix}$$

$$(M_{\pm}) \doteq \begin{pmatrix} M_2 & \sqrt{2}\, M_W \sin\beta \\ \sqrt{2}\, M_W \cos\beta & \mu \end{pmatrix}$$

- $M_2 \, \bar{\lambda}_{\tilde{W}} \lambda_{\tilde{W}}$
- $\mu \, \bar{\psi}_{\tilde{H}_1} \psi_{\tilde{H}_2}$
- MIXING



(4) NEUTRALINOS $\tilde{\chi}^0_{i=1,\dots,4}$

(New notation: $\tilde{N}_{i=1,\dots,4}$)

$\tilde{B}$ – bino $\quad U(1)_Y$

$\tilde{W}^0_3$ – wino $\quad SU(2)_L$

$\tilde{H}^0_1$ – HIGGSINO

$\tilde{H}^0_2$ – HIGGSINO

$\left.\right\}$ $S = \frac{1}{2}$, $Q = 0$ (colour $= 0$)

AFTER $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$ THESE CAN MIX

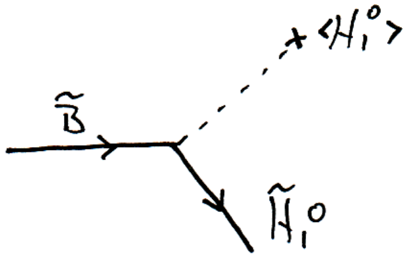
— AND THEY DO

$$(\tilde{B}, \tilde{W}^0_3, \tilde{H}^0_1, \tilde{H}^0_2)\,(M_0)\begin{pmatrix}\tilde{B}\\ \tilde{W}^0_3\\ \tilde{H}^0_1\\ \tilde{H}^0_2\end{pmatrix}$$

$$M_0 = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta s_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

Origin of terms:

- $M_1, M_2 \qquad \bar{\lambda}_{\tilde{B}} \lambda_{\tilde{B}}\ ,\ \bar{\lambda}_{\tilde{W}^0_3} \lambda_{\tilde{W}^0_3}$
- $\mu \qquad \dfrac{\delta^2 W}{\delta \tilde{H}^0_1 \delta \tilde{H}^0_2}\ \bar{\psi}_{\tilde{H}^0_1}\, \psi_{H^0_2}$
- MIXING : $\quad \bar{\psi} \lambda \varphi$



LIGHTEST NEUTRALINO: $\tilde{\chi}^0_1$ ($\tilde{N}^0_1$)

$\longrightarrow$ BEST CANDIDATE FOR DARK MATTER

$\boxed{\text{WHY?}}$

SUPERSYMMETRY – HIGGS SECTOR

(A) WHY 2 HIGGS DOUBLETS?

- SM: $Q H \bar{D} + Q \tilde{H}^* \bar{u}$

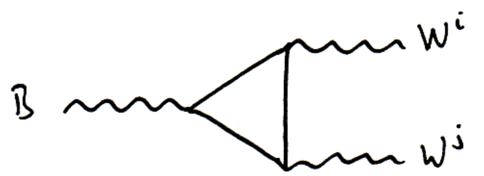
$$H = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad \tilde{H}^* = i\sigma^2 H^* = \begin{pmatrix} h_1^+ \\ h_1^{0*} \end{pmatrix}$$

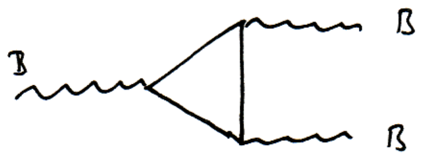
complex conjugate

IN SUPERPOTENTIAL: $W(\varphi)$, no φ^*!

$$\Rightarrow \quad \tilde{Q} H_1 \tilde{D}^c + \tilde{Q} H_2 \tilde{u}^c$$

- ANOMALIES: (QUANTUM CORRECTION WHICH VIOLATES SYMM. OF $\mathcal{L}$)





$\mathrm{Tr}(T_3^2 Y) = 0$ $\qquad\qquad$ $\mathrm{Tr}\, Y^3 = 0$

(Include colour factor!)

$$\left[\mathrm{Tr}(Y^3)\right]_{SM} = \underset{Q}{3\cdot 2\cdot \frac{1}{6\cdot 36}} \underset{u}{- 3\cdot\frac{8}{27}} \underset{d}{+ 3\cdot\frac{1}{27}} \underset{L}{- 2\cdot\frac{1}{8}} \underset{e}{+ 1} = 0$$

IN SUSY $H \longrightarrow \tilde{H}$ – fermion doublet, with $Y(\tilde{H}) = -\frac{1}{2}$

CONTRIBUTES TO ANOMALIES

$\Rightarrow$ NEED 2 HIGGS DOUBLETS $\quad H_1, H_2 \quad , Y(H_1) = -Y(H_2) = -\frac{1}{2}$

HOW MANY HIGGS FIELDS AFTER $SU(2) \times U(1)_Y \longrightarrow U(1)_{EM}$?

- SM: 1 COMPLEX DOUBLET $\Longrightarrow$ 4 REAL FIELDS

  3 FIELDS GET "EATEN" IN HIGGS MECHANISM

  $W^\pm, Z^0$ – GET MASS

  $\Rightarrow$ 1 FIELD LEFT $\boxed{h^0}$

- MSSM: 2 COMPLEX DOUBLETS $\Longrightarrow$ 8 REAL FIELDS

  $8-3=5$ REMAINING FIELDS

  h^0, H^0 CP-EVEN

  A^0 CP-ODD

  $H^\pm$

SUPERSYMMETRY HIGGS POTENTIAL

$$V = |F|^2 + |D|^2 + \text{SUSY-BREAKING}$$

$$F_i = \frac{\delta W}{\delta \varphi_i} \quad , \quad D^a = g\, \varphi_i^* (T^a)_{ij}\, \varphi_j$$

$$V = m_1^2 \left(|H_1^0|^2 + |H_1^-|^2 \right) + m_2^2 \left(|H_2^0|^2 + |H_2^+|^2 \right)$$

$$+ m_3^2 \left[\left(H_1^- H_2^+ - H_1^0 H_2^0 \right) + h.c. \right]$$

$$+ \frac{g^2 + g'^2}{8} \left[|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2 \right]^2 + \frac{g^2}{2} \left| H_1^{0*} H_2^+ + H_1^{-*} H_2^0 \right|^2$$

$$m_1^2 = \mu^2 + m_{H_1}^2$$

$$m_2 = \mu^2 + m_{H_2}^2$$

$$m_3^2 = B \cdot \mu$$

m_{H_1}, m_{H_2}, B: SOFT BREAKING PARAMETERS

(a) STABILITY OF POTENTIAL (V – FINITE FOR $H \to \infty$, neglect charged fields)

$$\Rightarrow \quad m_1^2 + m_2^2 \geq 2 |m_3|^2$$

(b) $SU(2) \longrightarrow$ CAN CHOOSE $\langle H_1^- \rangle = 0$

$$\Rightarrow \left(\frac{\partial V_H}{\partial H^-} = 0 \right) \Rightarrow \langle H^+ \rangle = 0$$

minimum condition

(a) $\boxed{m_1^2 + m_2^2 \geq 2|m_3|^2}$ $\Rightarrow$ m_1^2 or $m_2^2 > 0$

The other must be < 0 to get a vev.

$\Rightarrow$ SADDLE POINT IS ONLY NON-TRIVIAL MINIMUM

(c) $\det \left(\dfrac{\partial^2 V}{\partial H_i^0 \partial H_j^0} \right) < 0$

$\Rightarrow \boxed{m_1^2 m_2^2 < m_3^4}$

(a) & (c) CAN NOT BE SATISFIED IF $m_1^2 = m_2^2$

$m_1^2 - m_2^2 = m_{H_1}^2 - m_{H_2}^2$ SOFT-BREAKING

$\Rightarrow$ ONLY GET $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$

IF SUSY IS BROKEN

MINIMIZE HIGGS POTENTIAL (AFTER DROPPING H_1^-, H_2^+)

- SOLVE: $$\frac{\partial V}{\partial H_1^0} = \frac{\partial V}{\partial H_2^0} = 0$$

- USE $$\frac{g_1^2 + g_2^2}{2}(v_1^2 + v_2^2) = M_Z^2$$, KNOWN

- DEFINE: $$\tan\beta = \frac{\langle H_2^0 \rangle}{\langle H_1^0 \rangle} = \frac{v_2}{v_1}$$

$$\Rightarrow \quad m_1^2 = -m_3^2 \tan\beta - \frac{1}{2} M_Z^2 \cos(2\beta)$$

$$m_2^2 = -m_3^2 \cot\beta + \frac{1}{2} M_Z^2 \cos(2\beta)$$

$\boxed{m_3 \ \& \ \tan\beta}$ ONLY UNKNOWNS

DIAGONALIZE MASS MATRICES

$$\Rightarrow \quad m_{A^0} = -\frac{2m_3^2}{\sin(2\beta)}$$, REPLACE $m_3 \longrightarrow M_{A^0}$

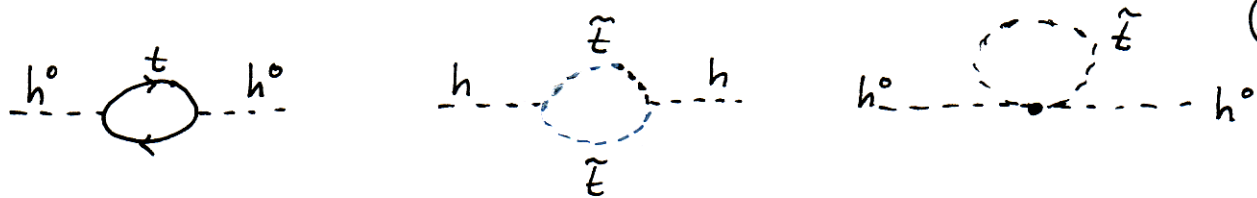
$$M^2_{H^0, h^0} = \frac{1}{2}\left(M_{A^0}^2 + M_{Z^0}^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2(2\beta)} \right)$$

$$\Rightarrow \quad \boxed{m_{h^0}^2 \leq M_Z^2}$$ WOW! (RECALL: $V \sim |D_a|^2 \leftarrow M_Z^2$)

BUT LEP II $\Rightarrow$ $\boxed{m_{h^0} \geq 113 \text{ GeV}}$

IS SUSY DEAD?

<u>No</u>



$$\Delta m_{h^0} = \frac{3 m_t^4}{32\pi^2 \sin^2\beta \, M_W^2} \log\left(\frac{m_{\tilde{t}_1}^2 \, m_{\tilde{t}_2}^2}{m_t^2}\right)$$

$\Longrightarrow \boxed{M_{h^0} \lesssim 135 \text{ GeV}}$ FOR $M_{SUSY} \lesssim \mathcal{O}(1 \text{ TeV})$

RAD. CORR. $\longrightarrow$

R-PARITY

RECALL:

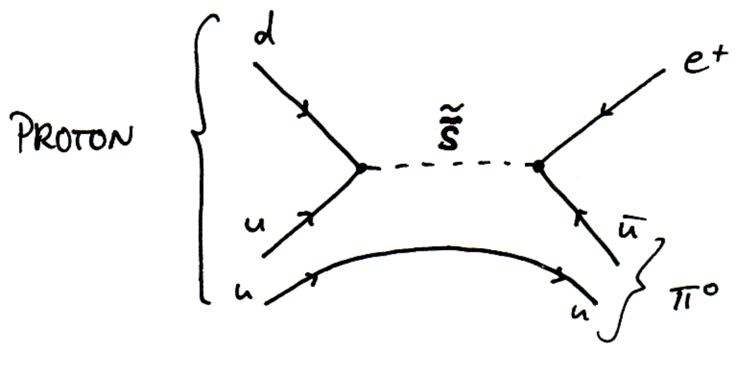
$$W_{SUSY-SM} = h^E_{ij}\,\tilde{L}_i H_1 \tilde{e}^c_{Rj} + h^D_{ij}\,\tilde{Q}_i H_1 \tilde{d}^c_{Rj} + h^u_{ij}\,\tilde{Q}_i H_2 \tilde{u}^c_{Rj} + \mu H_1 H_2$$

$$+ \underbrace{\lambda_{ijk}\,\tilde{L}_i \tilde{L}_j \tilde{e}^c_{Rk} + \lambda'_{ijk}\,\tilde{L}_i \tilde{Q}_j \tilde{d}^c_{Rk}}_{\Delta L \neq 0} + \underbrace{\lambda''_{ijk}\,\tilde{u}^c_{Ri} \tilde{d}^c_{Rj} \tilde{d}^c_{Rk}}_{\Delta B \neq 0} + \underbrace{\alpha_i \tilde{L}_i H_2}_{\Delta L \neq 0}$$

BUT: LEPTON- & BARYON-NUMBER VIOLATION HAVE NOT BEEN OBSERVED

$\xrightarrow{\text{bounds on } (\lambda, \lambda', \lambda'')}$

STRICTEST BOUNDS ON PRODUCTS: $\lambda'_{112}\,\tilde{L}_e \tilde{Q}_1 \tilde{s}^c_R + \lambda''_{112}\,\tilde{u}^c_R \tilde{d}^c_R \tilde{s}^c_R$



PROTON DECAY

$\Rightarrow \quad \lambda'_{112} \cdot \lambda''_{112} <$

$\Rightarrow \quad \lambda'_{112}$ OR $\lambda''_{112} = 0$, BY A SYMMETRY ?

MOST COMMON CHOICE: R- PARITY

$$R_p = (-1)^{2S+L+3B}$$

DISCRETE, MULTIPLICATIVE SYMMETRY

S: SPIN

L: LEPTON NUMBER

B: BARYON NUMBER

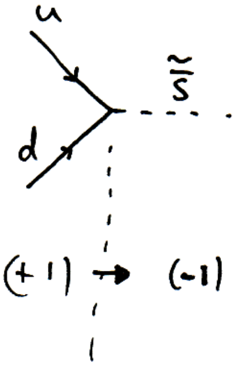
$$R_p(e^-) = (-1)^{2\cdot\frac{1}{2}+1+0}\, e^- = (+1)\cdot e^-$$

$$R_p(\tilde{e}^-) = (-1)^{2\cdot 0+1+0}\, \tilde{e}^- = (-1)\, \tilde{e}^-$$

ALL SUSY PARTICLES HAVE $R_p = -1$

$$R_p(\tilde{e}^-\tilde{e}^-) = (-1)^2 (\tilde{e}^-\tilde{e}^-) = (+1) \; (\tilde{e}^-\tilde{e}^-)$$ – multiplicative

VERTEX:



$R_p = \; (+1) \rightarrow (-1)$

VIOLATES R-PARITY

$\Rightarrow$ DEMAND R-PARITY (BY HAND) $\Rightarrow \underbrace{\tilde{L}\tilde{L}\tilde{E},\ \tilde{L}\tilde{Q}\tilde{D},\ \tilde{U}\tilde{D}\tilde{D},\ \tilde{L}\tilde{H}}_{\text{FORBIDDEN}}$

THIS IS THEN THE MSSM:

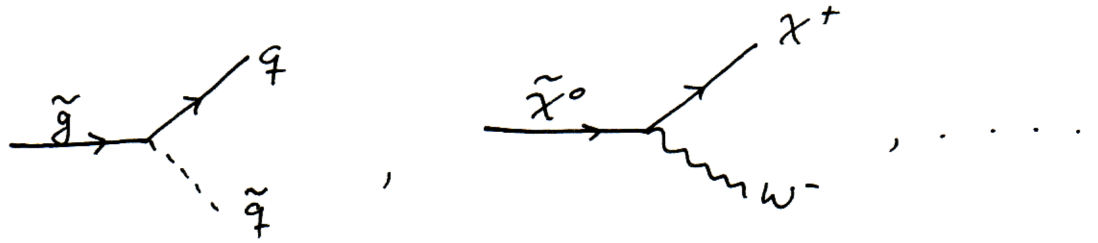
PARTICLE CONTENT: SM + SUSY + EXTRA HIGGS DOUBLET

GAUGE GROUP: $SU(3)_c \times SU(2)_L \times U(1)_Y$

SUPERPOTENTIAL:

$$W_{MSSM} = h^E_{ij} \tilde{L}_i H_1 \tilde{e}^c_{jR} + h^D_{ij} \tilde{Q}_i H_1 \tilde{d}^c_{Rj} + h^u_{ij} \tilde{Q}_i H_2 \tilde{u}^c_{Rj} + \mu H_1 H_2$$

NOTE: ALL OTHER VERTICES ALSO CONSERVE R_p!



ALWAYS CHANGE $\underline{\underline{2}}$ FROM SM-VERTEX.

$\Rightarrow$ WHOLE $\mathcal{L}$ CONSERVES R-PARITY

ASSUME: $\tilde{\chi}^0_1$ – lightest supersymmetric particle (LSP)

IT CAN NOT DECAY! IT IS R_p-ODD, BUT

ALL INTERACTIONS ARE R_p-EVEN.

$\Rightarrow$ $\tilde{\chi}^0_1$ – STABLE $\longrightarrow$ DARK MATTER?

R-Parity Violation & Neutrinos

If we choose Baryon-parity instead of R-parity:

$$B_p = (-1)^{2S+3B}$$

$$\Rightarrow W_{\not{R}_p} = \lambda_{ijk} \tilde{L}_i \tilde{L}_j \tilde{e}^c_{Rk} + \lambda'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{d}^c_{Rk} + \kappa_i \tilde{L}_i H_2$$

CONSIDER:

$$\frac{\delta^2 W_{\not{R}_p}}{\delta \tilde{L}_i \, \delta H_2} \bar{\psi}_{\tilde{L}_i} \psi_{H_2} = \kappa_i \bar{\psi}_{\tilde{L}_i} \psi_{H_2}$$

$$= \kappa_i \left(\nu_i \tilde{H}_2^0 - e_i \tilde{H}_2^+ \right)$$

NEUTRINOS & HIGGSINOS MIX

$\Rightarrow$ WE NOW HAVE A 7×7 NEUTRALINO MASS MATRIX

$$(\tilde{B}, \tilde{W}_3^0, \tilde{H}_1^0, \tilde{H}_2^0, \nu_e, \nu_\mu, \nu_\tau) \begin{pmatrix} M \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}_3^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

- SNEUTRINOS & HIGGS ALSO MIX

$$M = \begin{pmatrix} M_1 & 0 & M_Z s_W \frac{v_2}{v} & -M_Z s_W \frac{v_\beta}{v} \\ 0 & M_2 & -M_Z c_W \frac{v_2}{v} & M_Z c_W \frac{v_\beta}{v} \\ M_Z s_W \frac{v_2}{v} & -M_Z c_W \frac{v_2}{v} & 0 & -\mu_\beta \\ -M_Z s_W \frac{v_\alpha}{v} & M_Z c_W \frac{v_\alpha}{v} & -\mu_\alpha & 0_{\alpha\beta} \end{pmatrix}$$

$$v = \sqrt{v_1^2 + v_2^2 + \langle\tilde{\nu}_e\rangle^2 + \langle\tilde{\nu}_\mu\rangle^2 + \langle\tilde{\nu}_\tau\rangle^2}$$

$$v_\alpha = (v_1, \langle\tilde{\nu}_e\rangle, \langle\tilde{\nu}_\mu\rangle, \langle\tilde{\nu}_\tau\rangle)$$

$$\mu_\alpha = (\mu, \varkappa_e, \varkappa_\mu, \varkappa_\tau)$$

- DIAGONALIZE M $\Rightarrow$ 5 MASSIVE STATES

  $+ 2$ MASS-LESS $\rightarrow$ NEUTRINOS

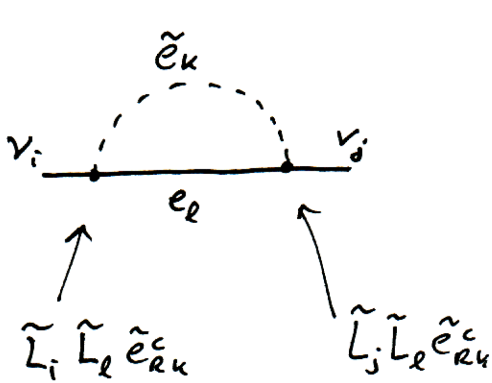
- 5 MASSIVE $\longrightarrow$ 4 NEUTRALINOS $+ 1$ NEUTRINO

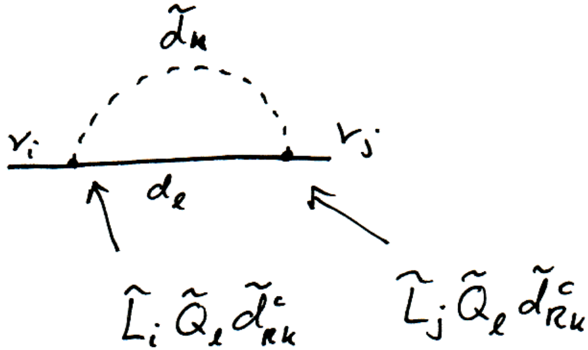
- CHOOSE $\varkappa_{e,\mu,\tau}$ SUCH THAT MASSIVE NEUTRINO $= \nu_\tau$

  & ATMOSPHERIC ν-PROBLEM

- MAKE TWO OTHER NEUTRINOS MASSIVE BY RAD. CORR.

$\longrightarrow$





$\Rightarrow$ CHOOSE λ, λ' to get correct spectrum + mixing.

<u>AMAZING</u>: DO NOT NEED ν_R !

DO NOT NEED SEE-SAW

OR NEW SCALE !