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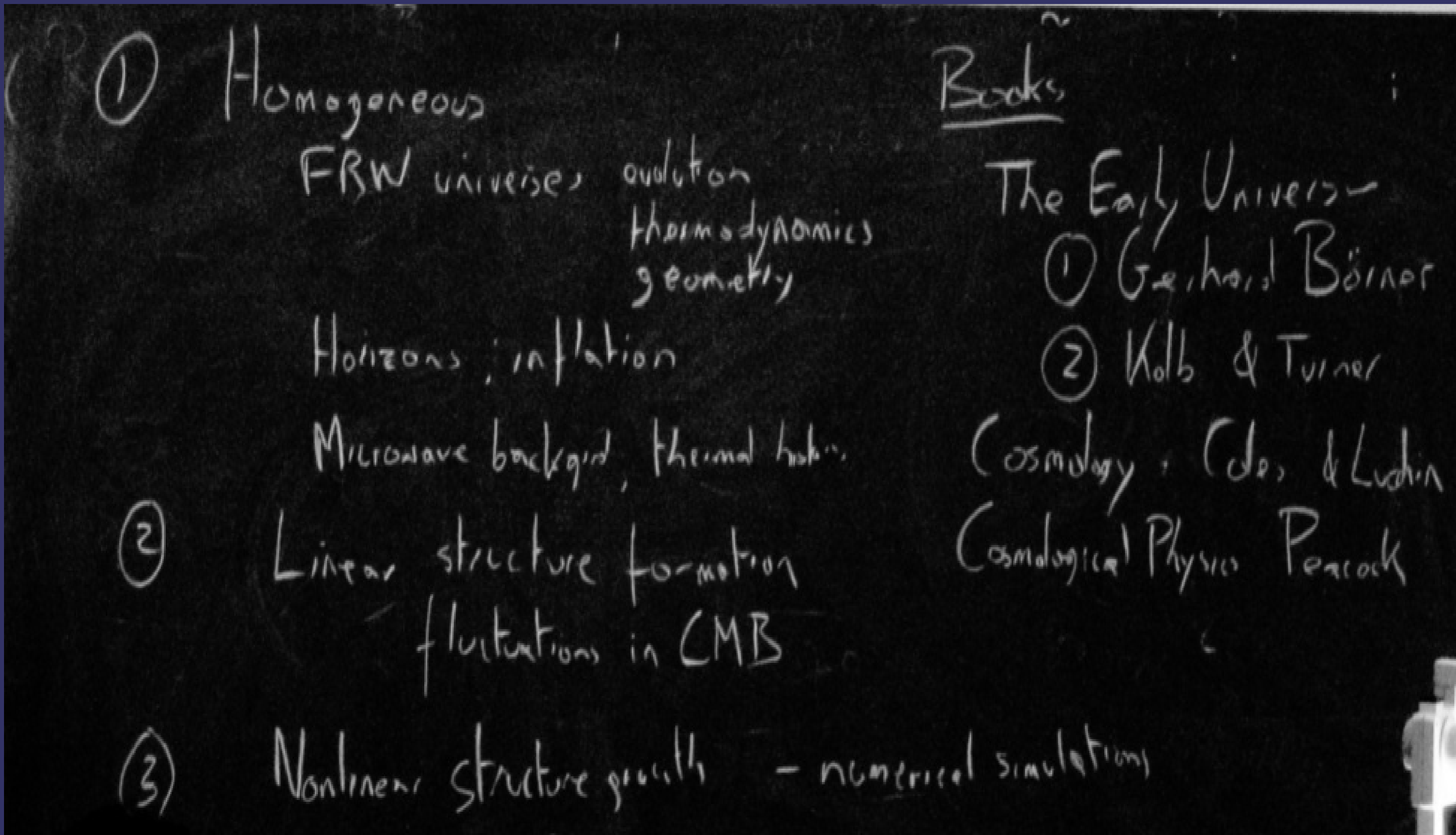
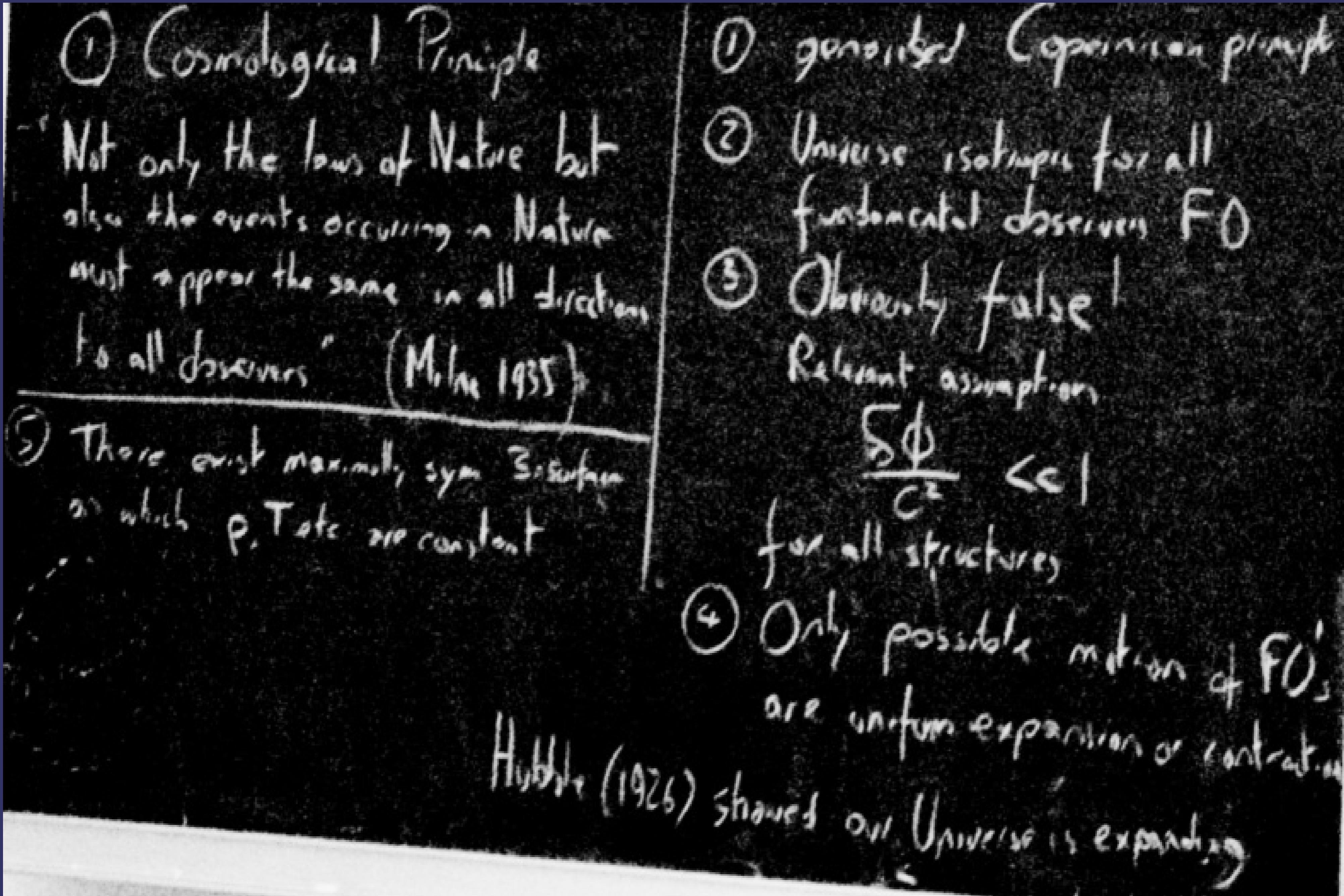
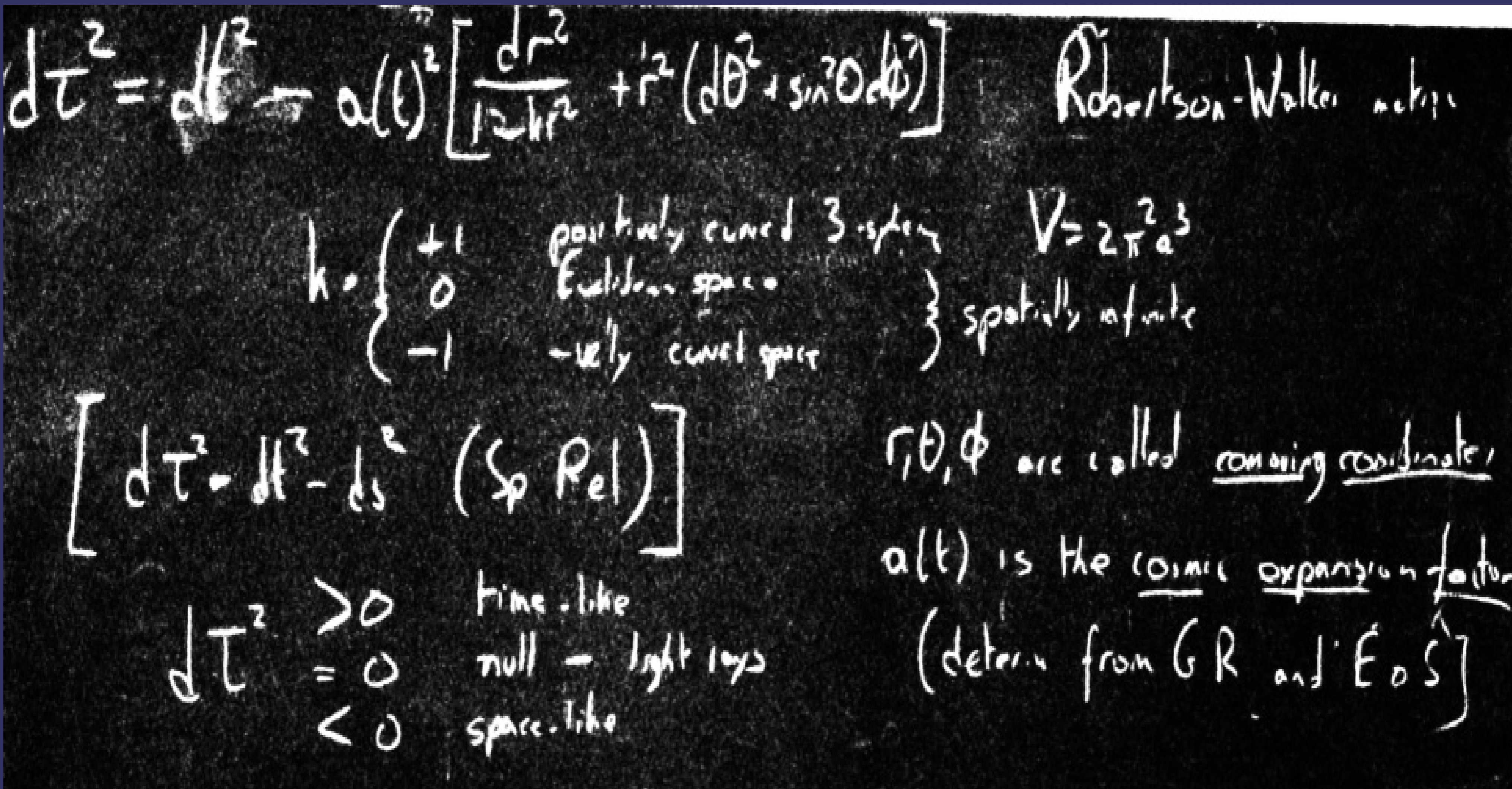
Cosmology and structure formation



① Homogeneous
FRW universes evolution
thermodynamics
geometry
Horizons ; inflation
Microwave backgrd, thermal hist…
② Linear structure formation
fluctuations in CMB
③ Nonlinear structure growth – numerical simulations
Books
The Early Universe –
① Gerhard Börner
② Kolb & Turner
Cosmology : Coles & Lucchin
Cosmological Physics Peacock


① Cosmological Principle
"Not only the laws of Nature but also the events occurring in Nature must appear the same in all directions to all observers" (Milne 1935)
⑤ There exist maximally sym 3-surfaces on which ρ, T etc are constant
① generalized Copernican principle
② Universe isotropic for all fundamental observers FO
③ Obviously false!
Relevant assumption
$\frac{\delta\Phi}{c^2} \ll 1$
for all structures
④ Only possible motion of FO's are uniform expansion or contraction
Hubble (1926) showed our Universe is expanding


dτ² = dt² − a(t)² [dr² / (1−kr²) + r² (dθ² + sin²θ dφ²)]
Robertson-Walker
k = +1 positively curved 3-space
0 Euclidean space
−1 −ve'ly curved space
V = 2π²a³
spatially infinite
[dτ² = dt² − ds² (Sp Rel)]
dτ² > 0 time-like
= 0 null − light rays
< 0 space-like
r, θ, φ are called comoving coordinates
a(t) is the cosmic expansion factor
(determ from GR and EoS)

(2) Free particle motions / thermodynamics

(1) massive nonrelat. particles

particles are unaccelerated w.r.t. their local FO. Hence their peculiar velocity satisfies

$$\delta v_p + (v_p \delta t)\frac{\dot a}{a} = 0 \Rightarrow v_p \propto \frac{1}{a}$$

Thus for a non-rel gas $\rho \propto a^{-3}$ $T \propto a^{-2}$

$P \propto \rho T \propto \rho^{5/3}$ $\quad P \ll \rho c^2$

More generally $dU = -p\,dV$ $\quad U = \rho V \propto a^3$

$$\rightarrow \frac{d\rho}{da} + 3\frac{p+\rho}{a} = 0 \quad \leftarrow$$

$$d\tau^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Robertson-Walker metric

(ii) Rel particles, photons

Consider light signal from a FO at r_e propagating to the origin

$$\int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_e+\delta t_e}^{t_o+\delta t_o} \frac{dt}{a(t)}$$

$$\longrightarrow \frac{\delta t_o}{a(t_o)} = \frac{\delta t_e}{a(t_e)} \longrightarrow \text{period} \propto \lambda$$

wavelengths of all photons are redshifted by

Take t_o to be the present

This gives standard redshift def

$$(1+z) = a(t_o)/a(t_e)$$

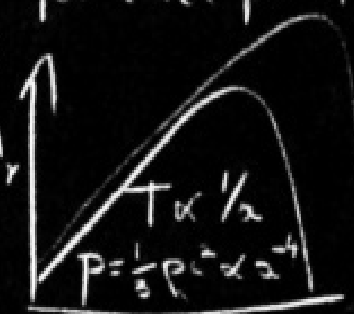
For a photon gas,

$\lambda \propto a$ for each photon

$n_\gamma \propto a^{-3}$

For a black body

$n_\gamma \propto T^3$ $\lambda_{peak} \propto 1/T$



$\longrightarrow$ BB radiation remains BB as U expands

② Free particle motions / thermodynamics

(ii) False vacuum / cosmological constant

non rel	$p = 0$
rel	$p = \frac{1}{3}\rho$
Cosmol. const	$p = -\rho$

$$\frac{d\rho}{da} + 3\,\frac{p+\rho}{a} = 0$$

If we have $\rho = \text{const}$ (e.g. false vacuum)

$$\rightarrow p = -\rho$$

More generally people write $p = w\rho$ where $w = -1$ is the cosmol. constant

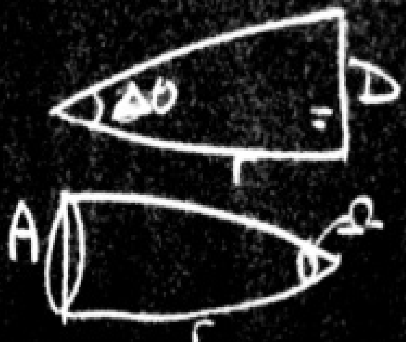
quintessence models $w > -1$ and time variable

$$d\tau^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$ Robertson-Walker metric

③ Angles & luminosities

Consider a source of proper size is D and intrinsic luminosity is L

$$D = a(t)\, r\Delta\theta = \frac{a(t_0)\, r}{1+z}\Delta\theta$$



Similarly an Area A at the origin subtends at source a solid angle

$$A = (a(t_0)\, r)^2\, \Omega$$

→ Received flux/unit area $f = \frac{1}{A}\frac{\Omega}{4\pi} L \left[\frac{a(t)}{a(t_0)}\right]^2 = \frac{L}{4\pi\left[a(t_0)\, r\,(1+z)\right]^2}$

One power of $\frac{a(t_0)}{a(t)}$ from photon arrival rate and second from photon energy redshift

$a(t_0)r/(1+z)$ are known as angular size distance

$a(t_0)r(1+z)$ luminosity

Observed surface brightness is

$$\frac{f}{\pi\Delta\theta^2} = \frac{L}{4\pi^2 D^2}(1+z)^{-4}$$

This dimming is indep of the relation between $a(t_0)r$ and z

(4) Relativistic Cosmologies

Einstein equations $$R_{\mu\nu} = -8\pi G\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}\right)$$

$R_{\mu\nu}$ is Ricci curvature tensor, $g_{\mu\nu}$ is the metric tensor

$T_{\mu\nu}$ is energy momentum tensor of matter/radiation.

$$d\tau^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)\right]$$ Robertson-Walker metric

Assume (i) RW metric (ii) isotropic perfect fluids

$$\rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$ Friedmann

With an EoS and an I.C. this specifies the evolution of $a(t)$

Differentiating once w.r.t. time and using 1st Law of TD

$$\ddot{a} = -\frac{4\pi}{3}G(\rho + 3p)a \qquad p = w\rho$$

Newtonian Eq of Motion for a uniform, uniformly expanding, pressure free sphere of active mass density $\rho + 3p$

Expansion accelerates / decelerates for $\rho + 3p \lessgtr 0$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_{vac} + \rho_{matter} + \rho_{radiation}\right) - \frac{k}{a^2}$$

Define $H = \frac{\dot{a}}{a}$ the Hubble parameter $H_0 \simeq 72 \pm 8$ km/s/Mpc

$\rho_{crit} = 8\pi G / 3H^2$ critical density $\rho_{crit,0} \sim 10^{-29}$ gm/cm^3

$\Omega = \rho/\rho_{crit}$ density parameters for matter, radiation, vacuum

$$H^2 = H^2\Omega_v + H^2\Omega_m + H^2\Omega_r - k/a^2$$

$$= H_0^2\Omega_{v_0} + H_0^2\Omega_{m_0}(1+z)^3 + H_0^2\Omega_r(1+z)^4 - \frac{k}{a_0}(1+z)^2$$

$\longrightarrow \frac{\dot{a}}{a}$ as a function of $1+z = \frac{a_0}{a} \longrightarrow a(t)$