Simon White

Cosmology and structure formation

Part II

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_v + \rho_m + \rho_r\right) - \frac{k}{a^2}$$ Friedmann equation

$$H^2 = H^2\Omega_v + H^2\Omega_m + H^2\Omega_r - \frac{k}{a^2}$$

$$= H_0^2\Omega_{v,0} + H_0^2\Omega_{m,0}(1+z)^3 + H_0^2\Omega_{r,0}(1+z)^4 - \frac{k}{a_0^2}(1+z)^2$$

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

5 Radiation-dominated Universe

At high z

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho_{r,0}\left(\frac{a_0}{a}\right)^4$$

$$\frac{a}{a_0} = \frac{1}{1+z} = \frac{T_0}{T(z)} = \left(\frac{8\pi G\rho_{r,0}}{3}\right)^{1/4}(2t)^{1/2}$$

Today $\rho_{r,0}$ is dominated by CMB $\rho_{r,0} \simeq 4.7\times 10^{-34}\ \mathrm{g/cm^3}$

(BB at $T = 2.73\,K$)

At early times ($z > 10^4$)

$$\frac{T}{10^{10}K} \sim \frac{kT}{1\,\mathrm{MeV}} \sim \frac{1+z}{10^{10}} \sim \left[\frac{\rho_{tot}}{10^7\,\mathrm{g/cm^3}}\right]^{1/4} \sim \left[\frac{\rho_{baryon}}{1\,\mathrm{g/cm^3}}\right]^{1/3} \sim \left[\frac{t}{1\,\mathrm{sec}}\right]^{-1/2}$$

Hot Big Bang

(6) Matter-dominated Universe

. Measurements of distance - redshift rel $\Rightarrow H_0 = 72 \pm 8$ km/s/Mpc

$\rightarrow \rho_{crit,0} = \frac{3H_0^2}{8\pi G} = 1.0 \times 10^{-29}$ g/cm^3 $H_0^{-1} = 14$ Gyr

$\rightarrow \Omega_{r,0} = 5 \times 10^{-5}$ $\nearrow \rho_{m,0} \simeq 3 \times 10^{-30}$ g/cm^3

CMB observations suggest $\Omega_{tot} \simeq 1$, $\Omega_{m,0} \simeq 0.3$, $\Omega_{v,0} \simeq 0.7$

Universe switched from rad dom $\rightarrow$ matter dom at $1+z_{eq} \simeq \frac{\Omega_{m,0}}{\Omega_{r,0}} \simeq 6000$

" " " matter dom $\rightarrow$ vacuum dom at $1+z_{vac} \simeq \left(\frac{\Omega_{v,0}}{\Omega_{m,0}}\right)^{1/3}$

7 Horizon

A light ray emitted by an event at (r, t_1) reaches the origin at t_2 where

$$\int_0^r \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{a_1}^{a_2} \frac{da}{\sqrt{\frac{8\pi G\rho a^2}{3} - k}\; a}$$

$\rho_r \propto a^{-4}$

$\propto e^{Ht}$

If the t (or a) integral converges as $t_1 \to 0$ ($a_1 \to 0$)

there exist FO who cannot yet have communicated with us

Such observers are beyond our particle horizon

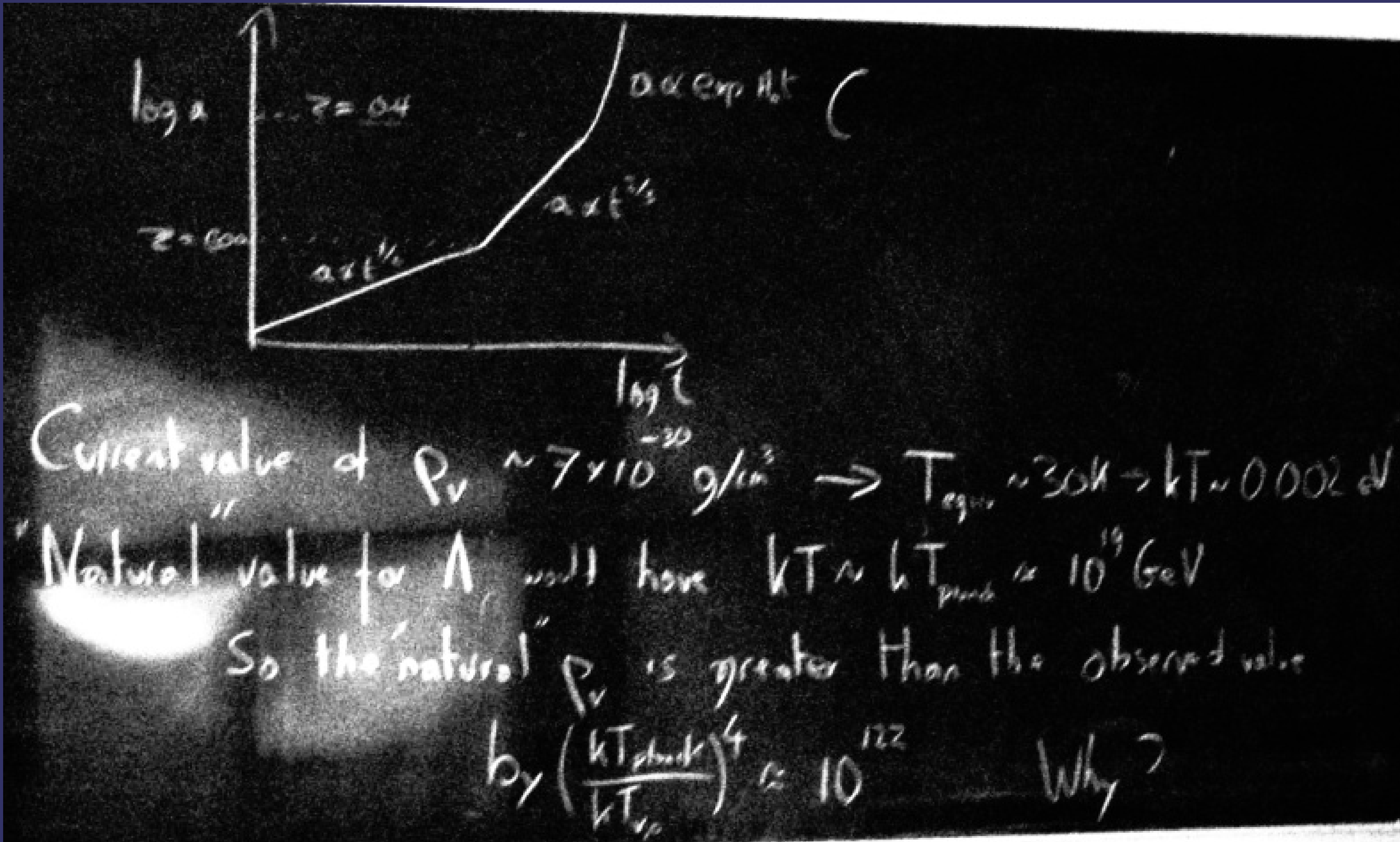
Not true for a vacuum dom universe

particle horizons expand with t_2

True for rad'n-dom universe

matter

Convergence requires $\rho a^2 \to \infty$ as $a \to 0$


log a
z = 0.4
a ∝ exp Λt
a ∝ t^2/3
a ∝ t^1/2
log t
Current value of ρv ~ 7×10^-30 g/cm^3 → Tequiv ~ 30K → kT ~ 0.002 eV
"Natural" value for Λ would have kT ~ kTplanck ~ 10^19 GeV
So the "natural" ρv is greater than the observed value
by (kTplanck / kTv)^4 ≈ 10^122
Why?

(7) Horizon

A light ray emitted by an event at (r, t_1) reaches the origin at t_2 where

$$\int_0^r \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{a_1}^{a_2} \frac{da}{\sqrt{\frac{8\pi G\rho a^2}{3} - k}\; a}$$

$\rho_r \propto a^{-4}$
$\propto e^{Ht}$

If the t (or a) integral converges as $t_1 \to 0$ ($a_1 \to 0$)

there exist FO who cannot yet have communicated with us

Such observers are beyond our particle horizon

Not true for a vacuum dom. universe

particle horizons expand with t_2

True for rad'n-dom universe, matter

Convergence requires $\rho a^2 \to \infty$ as $a \to 0$

(8) Inflation $t \sim 10^{-30}$ sec

At some early time $\left(kT \sim kT_{GUT}\right)$ the Universe passed through a stage when $\ddot{a} > 0$ $\left(\Longleftrightarrow \rho a^2 \text{ increases with } a\right)$ during which a increases by a large factor

e.g. pass through a GUT phase transition and get "stuck" in a phase with $\rho_{vac} > 0$ (and dominant) for a time δt such that

$$H \delta t \gg 1 \qquad H^2 = \frac{8\pi G \rho_v}{3}$$

Inflation ends when transition to new phase is made

(1) Flatness problem

Universe today is almost flat $\quad \Omega_0 = \Omega_{\Lambda,0} + \Omega_{m,0} + \Omega_{r,0} \simeq 1 \pm 2\%$

WMAP

$$H^2 = H^2\Omega_{tot} - k/a^2$$

$$\rightarrow \quad 1-\Omega_{tot} = k/H^2a^2$$

$$H \propto \begin{cases} a^{-2} & \text{rad dom} \\ a^{-3/2} & \text{matter dom} \end{cases} \quad \Rightarrow \quad 1-\Omega_{tot} \propto \begin{cases} a^{2} & \text{rad dom} \\ a & \text{matter dom} \\ a^{-2} & \text{vacuum dom} \end{cases}$$

At t_{Planck} $\quad kT_p \sim 10^{19}\,\text{GeV} \quad 1-\Omega \sim 10^{-58}$

Why is $1-\Omega_{t_{\rm Planck}}$ so close to zero

Why is the Universe so big? $\left(\text{cf } L_{\rm Planck} \sim 10^{-33}\ \text{cm} \right)$

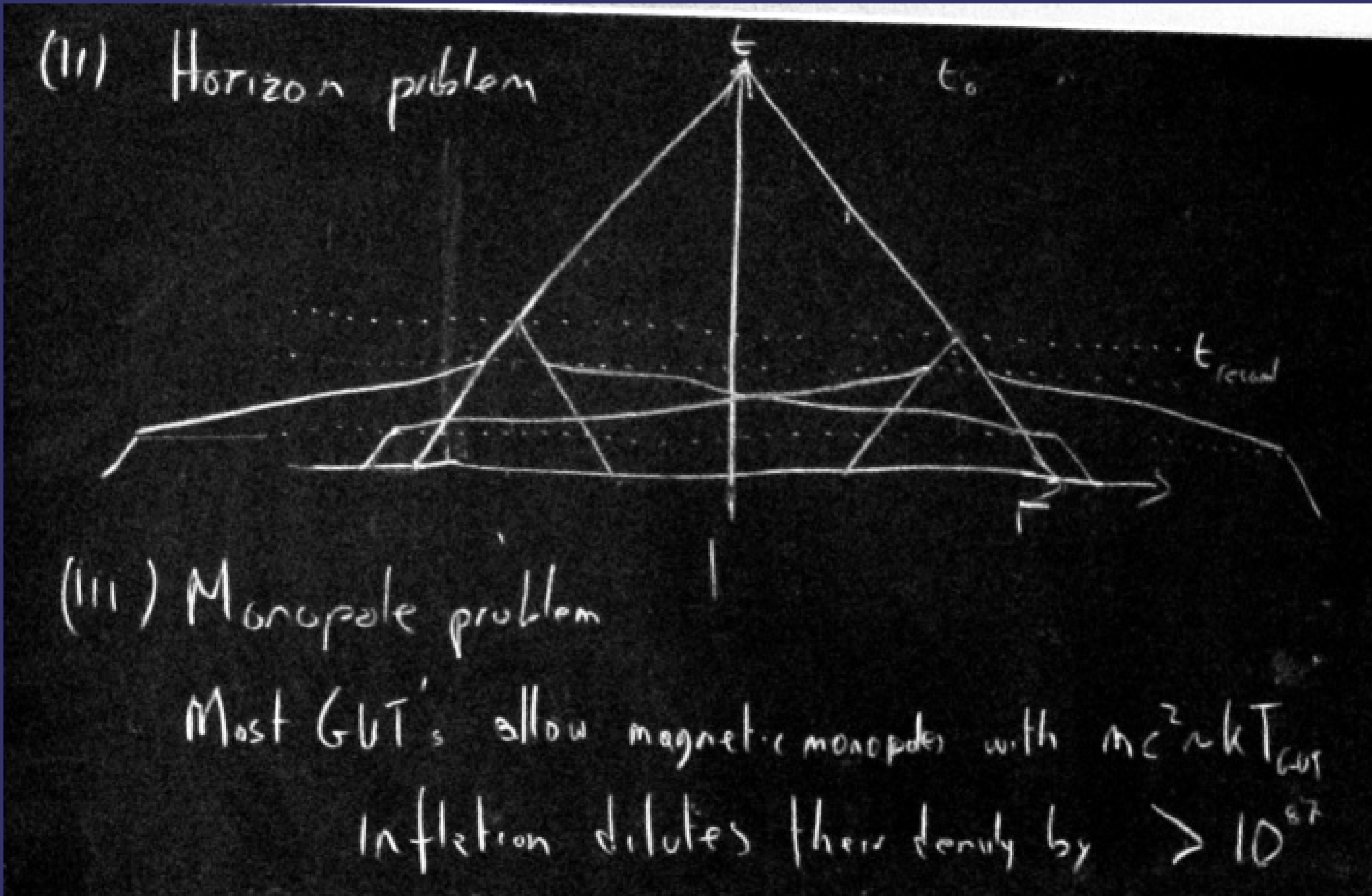
Why " " " so old? $\left(\text{cf } t_{\rm Planck} \sim 10^{-43}\ \text{s} \right)$

During inflation, however $1-\Omega \propto a^{-2}$

So if $H\,\delta t > 67$ then the expansion is $> e^{67} \sim 10^{29}$

If $1-\Omega$ is $O(1)$ at the beginning it is $< 10^{-58}$ at the end

If $H\,\delta t$ __exceeds__ 67 then $1-\Omega \approx 0$ today


(II) Horizon problem
t
t_0
t_{recomb}
r
(III) Monopole problem
Most GUT's allow magnetic monopoles with $mc^2 \sim kT_{GUT}$
Inflation dilutes their density by $> 10^{67}$