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Cosmology and structure formation

Part III

Problems "solved" by Inflation

(1) flatness (2) horizons (3) monopoles

(4) Structure Problem

For a galaxy cluster: mass $10^{15} M_\odot \sim 10^{72} m_p$

Quantum fluct'ns in a non-inflating universe cannot give this amplitude

$\frac{\delta\Phi}{c^2} \sim 10^{-5}$

$1/\sqrt{N} \sim 10^{-36}$

$a \sim e^{Ht}$ $\rho = \rho_{vac} = \frac{3H^2}{8\pi G} = \text{const}$

During inflation Universe is approx time-invariant

$\Rightarrow$ quantum fluctuation amplitudes are time-inv.

$\Rightarrow$ fluct'ns which "exit" the horizon at different times during inflation will have (statistically) the same amplitude, but scales which differ by

$$\frac{\lambda_1}{\lambda_2} \sim \exp\left[H(t_1 - t_2)\right]$$

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$t \sim 1/H$

$\lambda_{horiz} \sim c/H$

At a much later time $\lambda \frac{a(t)}{a_0} = ct$ the fluctuations re-enter the horizon with the same amplitude

If fluctuations originated as zero-point fluctuations of inflaton field → $\frac{\delta\phi}{c^2}$ const on all ~~scales~~ scales as fluct'ns cross horizon

Harrison Zel'dovich fluctuations

9) Thermal history of Universe

T	t	
10^{15} GeV	10^{-36} s	Possible epoch of inflation (?) Generation of structure
>100 MeV	$<10^{-4}$ s	"quark soup" no bound hadrons. baryogenesis (?)
~ 100 MeV	10^{-4} s	quark hadron phase transition $\rightarrow \gamma, \nu, e^{\pm}, \mu, p, n$: $n(\gamma) \sim 10^{9}\,(n(p)+n(n))$
~ 3 MeV	0.1 s	ν's "decouple" ie rate for $e^- + e^+ \leftrightarrow \nu + \bar{\nu}$ becomes long compared to $t \sim 1/H$ Universe becomes transparent to ν's

T	t	
~1 MeV	1 s	rates for $p + e^- \xleftrightarrow[cc]{} n + \nu_e$ $\quad n + e^+ \longleftrightarrow p + \bar{\nu}_e$ drop below H → final $\frac{n}{p} \sim \exp\left(\frac{-\Delta m}{kT}\right) \sim 20\%$
0.3 MeV	10 s	e^+e^- annihilation → heats photon gas establishes final $\frac{n_\gamma}{n_\nu} \sim \frac{11}{3}$ $\quad \frac{n_\gamma}{n_{baryon}} \sim 10^9$
0.1 MeV	100 s	$p + n \rightarrow D$ becomes possible. Almost all n are bound to ^4He
0.05 MeV	400 s	Nucleosynthesis complete 76% H, 24% ^4He + small amounts of ^2H ^3He ^7Li
		Exact amounts depend strongly on $\frac{n_{baryon}}{n_\gamma}$

(9) Thermal history of Universe

T	t	
3×10^{6} K	1 yr	Inelastic scattering of γ becomes inefficient Black-body spectrum cannot created *after* this time Observed CMB $\rightarrow$ no sign. input of γ after this time
20,000 K	8000 yr	Universe switches from radiation $\rightarrow$ matter domination.
3000 K	3×10^{5} yr.	Recombination rate pre $\rightarrow$ H dominates reionization by the Wien tail of the CMB photons. Recombination Universe becomes transparent to photons
2.73 K	13.2 Gyr	TODAY

Nonlinear growth of structure in the standard ΛCDM paradigm with inflationary fluctuations adjusted to fit the WMAP data on CMB fluctuations

Evolution of the dark matter distribution in a thin slice centred on a galaxy cluster at z=0

Zoom into a rich cluster from a very large-scale image of the z=0 DM distribution in a thin slice

Evolution of the DM and gas distributions, the X-ray and S-Z images of a rich galaxy cluster

(10) Structure formation

$$\rho(\underline{r},t) = \bar{\rho}(t)\left(1 + \delta(\underline{r},t)\right) \qquad \delta \ll 1$$

Linearise the evolution eqn's to get a linear equation for $\delta(\underline{r},t)$

FT the spatial dependence $\delta_k \sim \int d^3x\, \delta(\underline{r},t)\, e^{i\underline{k}\cdot\underline{x}}$

to get ODE's in time for $\delta_{\underline{k}}(t)$

at late times

Result At late times and or large scales

$$\delta_{\underline{k}}(t) = b(t)\,\delta_k(t_0) \qquad b \text{ indep of } \underline{k} \qquad b \propto t^{2/3} \propto a$$

⑨ Thermal history of Universe

- Characterise linear fluctuations by their power spectrum $P(k)$

- Inflationary fluct are completely characterised by $P(k)$

$$\rightarrow P(k) \propto k^n \quad \text{with } n \approx 1$$

Waves with diff k have uncorrelated phases $\rightarrow \delta$ is a gaussian random field

Shape of $P(k)$ at recombination depends

① Structure generator

② Nature of DM