Quantum Gravity in Extra Dimensions

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Work in collaboration with Daniel Litim

Motivation

- Quantum gravity

- Planck cosmology
  Quantum black holes

- Cosmological constant –
  Why this value?
  Why now?

- X-tra dimensions
  Planck physics at TeV in LHC
  How does Gravity look in them?

Asymptotic Safety

- Weinberg 1979: non-perturbative renormalizability

  perturbative renormalizability
  $\Rightarrow$ trivial UV fixed point (e.g. QCD)

  non-trivial UV fixed point $\Rightarrow$ non-perturbative renormalizability (e.g. Gross-Neveu)

- Gravity

  Weinberg 1979: UVFP perturbatively in $2 + \varepsilon$ dimensions

  Recent progress in 4d gravity:
  Reuter (1996); Souma (1999) ;
  Lauscher, Reuter (2000), (2001);
  Forgacs, Niedermaier (2002);
  Perini, Percacci (2003);
  Litim (2003); Hamber (2000) (lattice)

Wilsonian Renormalization Group

Exact Renormalization Group Flow

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} Tr \frac{1}{\Gamma_k^{(2)}[\phi] + \mathcal{R}_k} \partial_k \mathcal{R}_k$$

- Effective action $\Gamma_k[\phi]$
  Field modes ϕ_q with $q > k$ have been integrated out

- $\mathcal{R}_k$ provides the momentum cutoff

- Implementation for quantum gravity by Reuter (1996)

Einstein Hilbert Gravity

Truncation

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^d x \sqrt{g} \big(R - 2\bar{\lambda}_k \big)$$

Canonical dimensions

$$[G_k] = 2 - d \quad , \quad [\bar{\lambda}_k] = 2$$

Renormalized dimensionless couplings

$$g_k = G_k \, k^{d-2} \quad , \quad \lambda_k = \bar{\lambda}_k \, k^{-2}$$

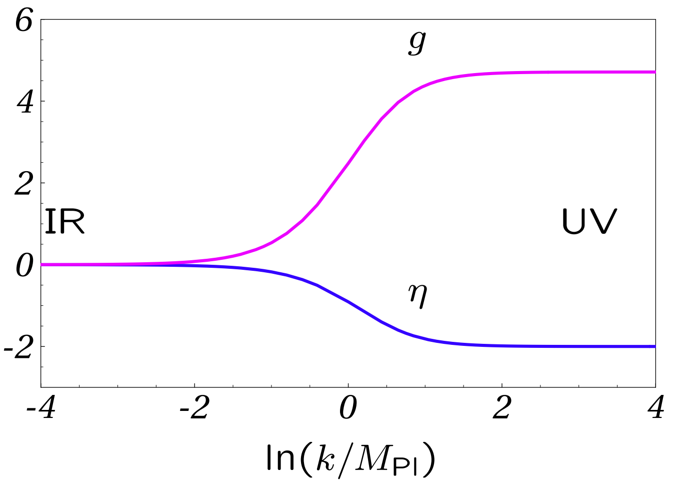
Renormalization group equations

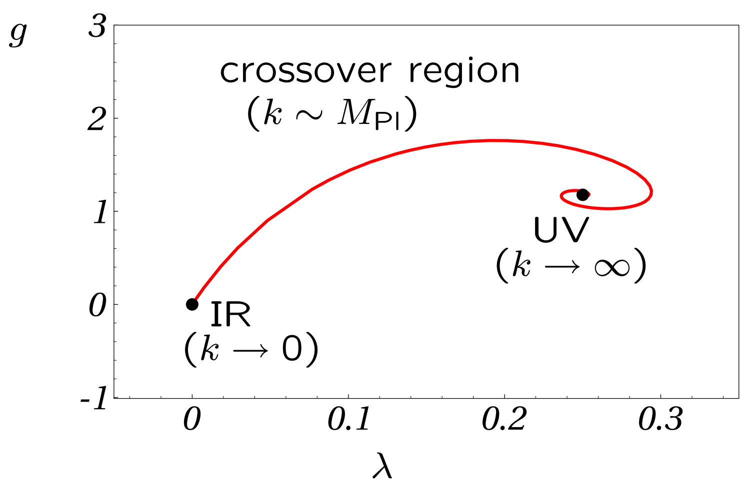
$$k \, \partial_k g \equiv \beta_g \quad , \quad k \, \partial_k \lambda \equiv \beta_\lambda$$

$$\beta_g = (2 - d - \eta) g$$

4d Flow

Example



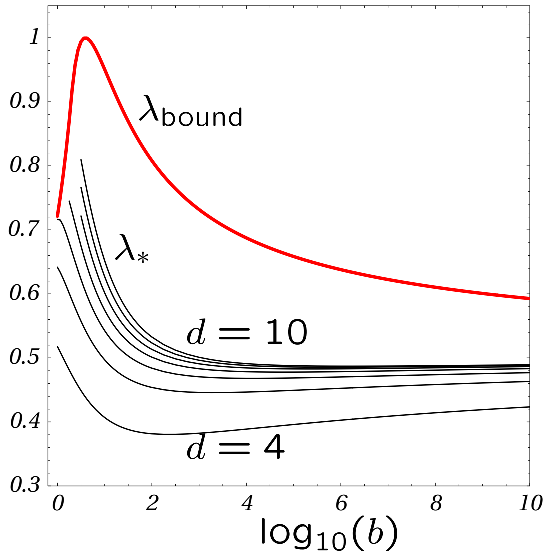


Extra Dimensions

Fixed point analysis

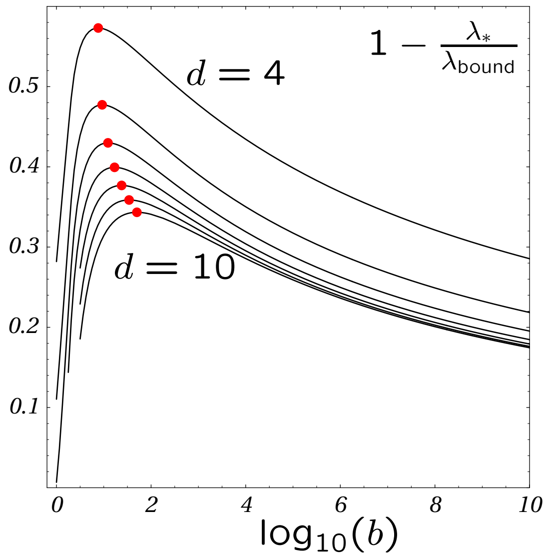
Solve $\beta_g(g_*, \lambda_*) = 0 \quad , \quad \beta_\lambda(g_*, \lambda_*) = 0$

All solutions obey $\lambda_* > 0\,, \quad g_* > 0$



$$\mathcal{R} = \frac{q^2 b}{(b+1)^{q^2/k^2} - 1}$$

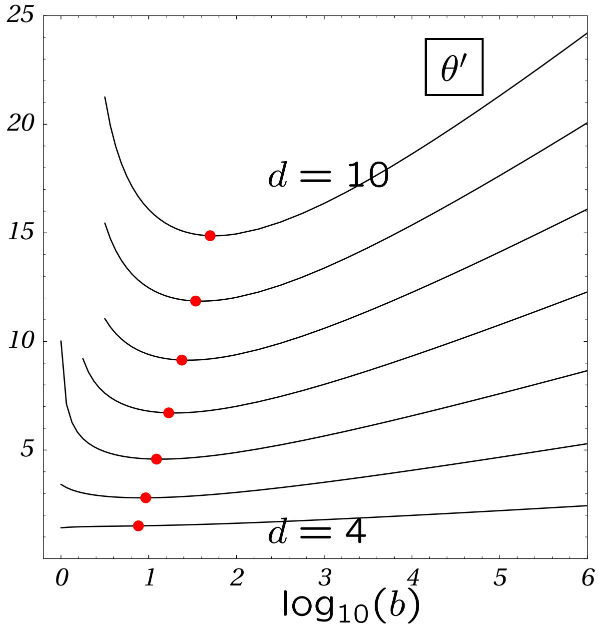
Stability

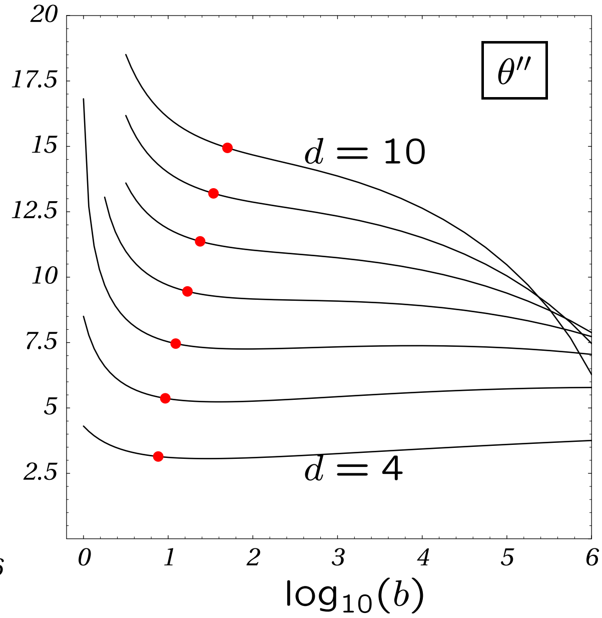


Universality

Fixed points are non-universal

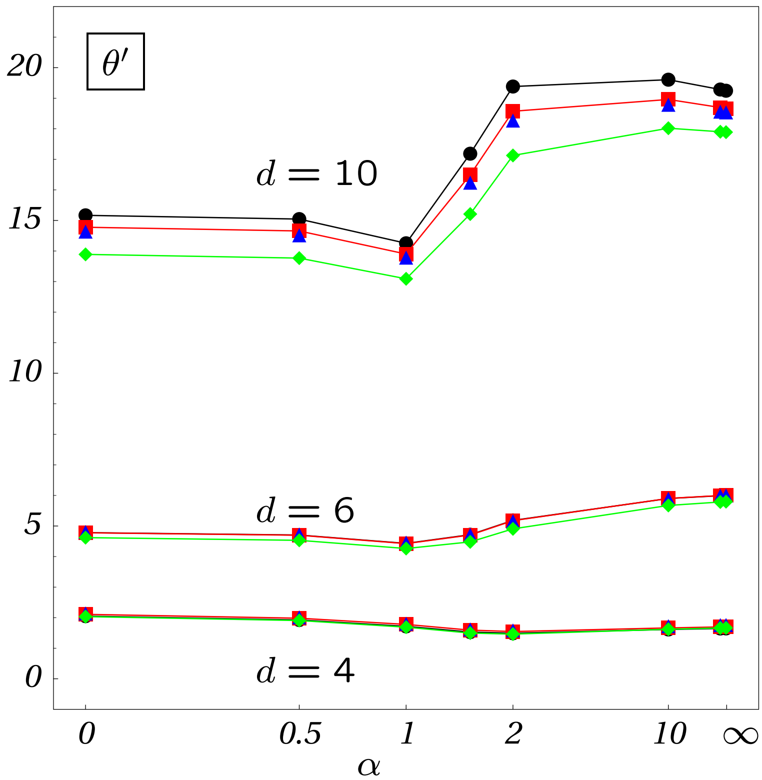
Eigenvalues of stability matrix $\theta = \theta' \pm i\theta''$





Gauge-fixing Independence

Compare optimised results from different cut-offs and gauge fixings.



Conclusions

- Ultraviolet fixed points found for quantum gravity in extra dimensions

- Highest reliability of present truncation due to the underlying optimisation

- If this picture persists in all extended truncations, quantum gravity is asymptotically safe

- Implications for extra dimensional phenomenology at LHC

Exact Renormalization Group Equation

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} Tr \frac{1}{\Gamma_k^{(2)}[\phi] + \mathcal{R}_k} \partial_k \mathcal{R}_k$$

Full propagator $\Gamma^{(2)} \ldots \delta^2 \Gamma[\phi]/\delta^2 \phi$

Trace $Tr \ldots tr_{Spin} tr_{\phi_i} tr_{flavor} \int d^d q$

IR-cutoff $\mathcal{R}(q^2/k^2)$

$$\begin{aligned} q \gg k \quad &: \quad \mathcal{R}(q^2/k^2) \; \to 0 \\ q \ll k \quad &: \quad \mathcal{R}(q^2/k^2) \; \to \propto k^2 \end{aligned}$$

e.g. modified exponential $\mathcal{R} = \frac{q^2 b}{(b+1)^{q^2/k^2} - 1}$

generalized optimal $\mathcal{R} = b(k^2 - q^2)\Theta(k^2 - q^2)$

Extended Truncations

- Higher Truncations ($+R^2$): slight influence, EH very good approximation
  Lauscher, Reuter 2002

- Matter minimally added: if # fields small, FP remains attractive
  Percacci, Perini 2003

Other Applications

- Planck-time Cosmology: no Horizon Problem, flat R-fluctuation spectrum (spectral index $n = 1$)

- BH: evaporation stops, relics of M_{Pl}

- IRFP Cosmology: conjecture IRFP, predict near FP
$\Omega_\Lambda = \Omega_m = 1/2$

- by Bonanno, Reuter

Asymptotic Safety of Quantum Gravity

- S. Weinberg, in *General Relativity: An Einstein centenary survey*, Eds. S. W. Hawking and W. Israel, Cambridge University Press (1979), p.790

Renormalization Group and Gravity

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- S. Falkenberg and S. D. Odintsov, “Gauge dependence of the effective average action in Einstein gravity”, Int. J. Mod. Phys. A **13** (1998) 607 [arXiv:hep-th/9612019]

Applications to Cosmology, BHs

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