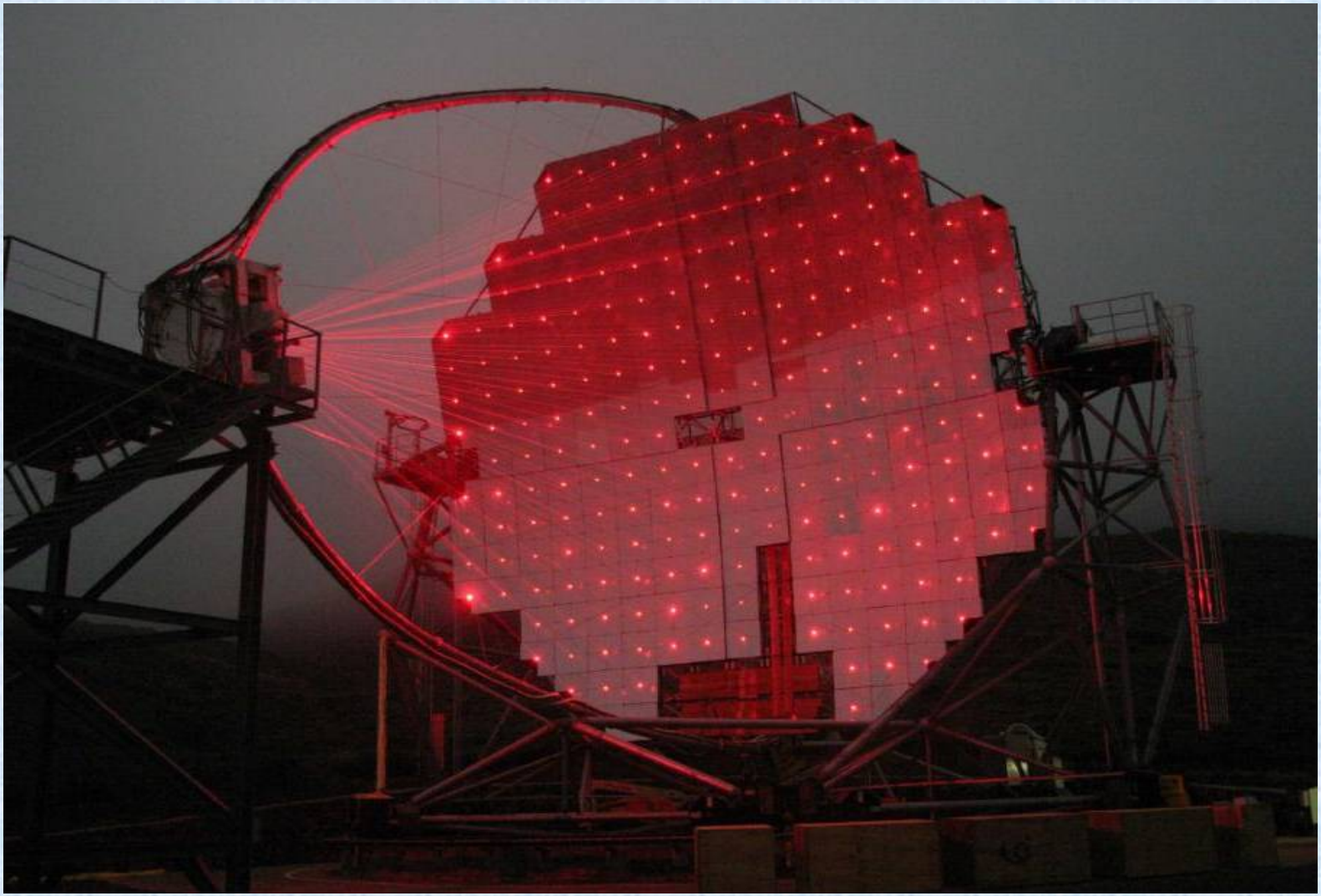
Fourier analysis on MAGIC



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The Standard-Method (MAGIC)

The Hillas parameters:

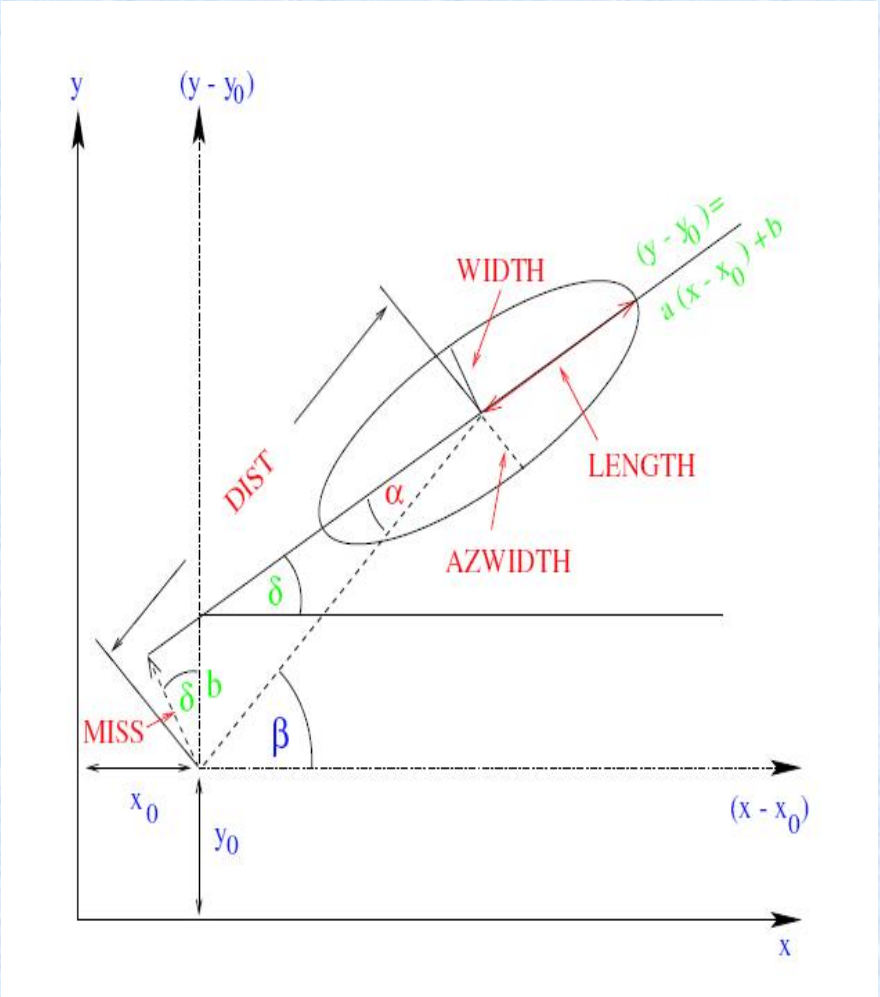
SIZE – total nr. of photons in the shower image.

LENGTH/WIDTH – RMS value of the light distribuition along the main/minor axis of the shower image.

DELTA – angle between the shower axis and the x-axis of the camera.

DIST – distance between the reference point and center of gravity (COG).

ALPHA – angle between the shower axis and the line connecting the COG of the shower axis and reference point.



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The Standard-Method (MAGIC)

Calculation of the moments and correlations:

$$
\begin{aligned}
w_i &= \frac{N_i}{\sum_k N_k} & \overline{x^2} &= \sum_i w_i \cdot x_i^2 \\
\overline{x} &= \sum_i w_i \cdot x_i & \overline{y^2} &= \sum_i w_i \cdot y_i^2 \\
\overline{y} &= \sum_i w_i \cdot y_i & \overline{xy} &= \sum_i w_i \cdot x_i \cdot y_i
\end{aligned}
$$

$$
\begin{aligned}
c_{xx} &= \overline{x - \overline{x}^2} = \overline{x^2} - \overline{x}^2 \\
c_{yy} &= \overline{y - \overline{y}^2} = \overline{y^2} - \overline{y}^2 \\
c_{xy} &= \overline{x - \overline{x}y - \overline{y}} = \overline{x \cdot y} - \overline{x} \cdot \overline{y}
\end{aligned}
$$

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The Standard-Method (MAGIC)

Calculation of the Hillas parameters:

$$\begin{aligned}
\mathrm{SIZE} &= \sum_k N_k \\
\mathrm{LENGTH} &= \sqrt{\frac{c_{xx}+2a\cdot c_{xy}+a^2\cdot c_{yy}}{1+a^2}} \\
\mathrm{WIDTH} &= \sqrt{\frac{a^2\cdot c_{xx}-2a\cdot c_{xy}+c_{yy}}{1+a^2}} \\
\mathrm{DIST} &= \sqrt{(\overline{x}-x)^2+(\overline{y}-y)^2} \\
\mathrm{ALPHA} &= \arcsin\left(\frac{|b|}{\mathrm{DIST}\cdot\sqrt{1+a^2}}\right)
\end{aligned}$$

$$a=\frac{c_{yy}-c_{xx}+\sqrt{c_{yy}-c_{xx}^2+4c_{xy}^2}}{2c_{xy}};\ b=(\overline{y}-y)-a\cdot((\overline{x}-x))$$

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The Standard-Method (MAGIC)

Data:

~ On – Off

~ Wobbel

~ On

Separation:

~ ALPHA cut

~ Tail cut, etc.

~ Optimization

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Fourier method (HEGRA)

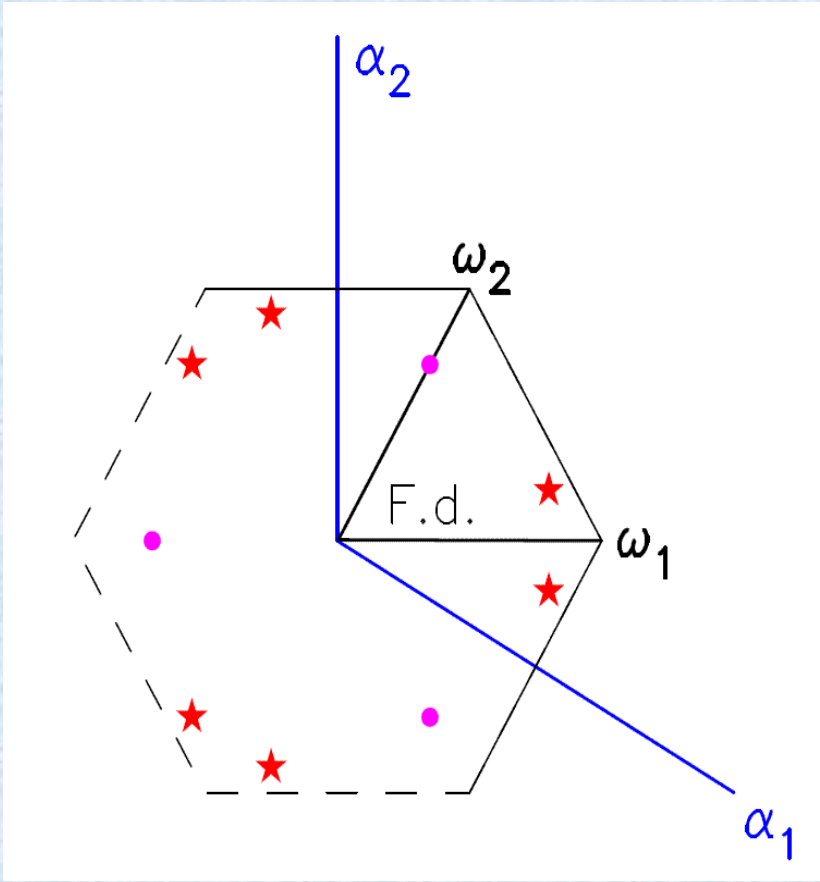
~ Transform the discrete image in wave number space, or frequency (i.e. transform the time argument).

~ Image is defined on a grid with hexagonal symmetry, which is a case of Fourier Transforation (FT) on the group SU(3) => Discret Group Transformation (DGT).

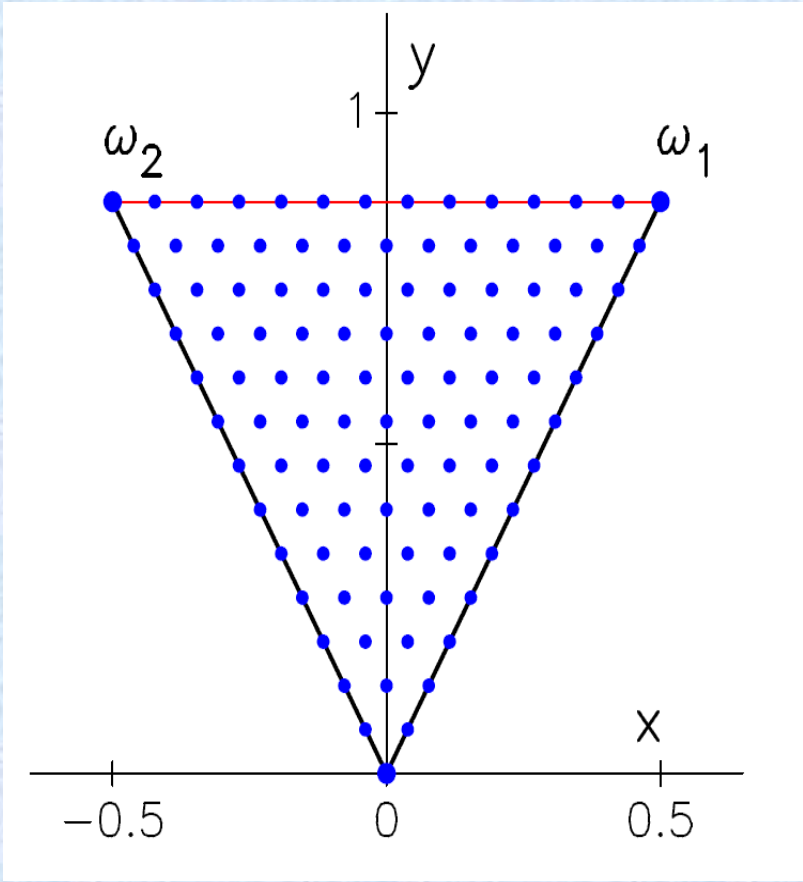
~ The advantage of DGT is that the Continuous Extension of DGT (CEDGT) converges to the grid function, but the Continuous Extension of DFT (CEDFT) not all the time.

~ Using the CEDGT we can calculate the coefficients of the FT.

Fourier method (HEGRA)



=>



Hexagonal grid of camera => Fundamental domain (FT)

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Fourier method (HEGRA)

~ We go from the hexagonal grid to fundamental domain.

~ Reconstraction of the photoelectric distribution.

~ Calculate the FT image, i.e. the coefficients, and those which do not pass the given filter (different cuts) are discarded from the series.

~ With the obtained CEDGT function the value of the brightness distribution can be calculated for a new grid.

~ The new grid can contain any higher density of points (due to the fact that the CEDGT is continuous).

~ The Fourier method delivers a better result as the Standard method just for a few pixels (PMTs), i.e. for low energy.

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How to connect...

The difference between MAGIC and HEGRA is about the form of the camera, i.e. MAGIC has two different PMTs sizes.

The problem is to find the fundamental domain for the new geometric form (i.e. of the camera).

Two options

~ to transform somehow the problem in terms of HEGRA;

... but how?

~ to calculate analytic the new fundamental domain of the group

... which group ?

How to connect...

First possibility:

A. To factorize the large PMTs with small PMTs.

Advantage: we get immediately the wished form of a hexagon.

Disadvantage: the factorization depends on PMTs, angle, etc, which means it is not a number, but a function.

B. To overgrid the large PMTs with small PMTs, still we have a hexagon over all the camera with small PMTs.

Advantage: increasing of signal of the small PMTs by adding from large PMTs in the neighbourhood.

Disadvantage: change of the inital form of the shower .

How to connect...

Second possibility:

To calculate the fundamental domain of a geometric form of the camera with two different characteristics (the different PMTs) we need to do this for both of them separately.

To connect the two obtained fundamental domains we need a group in two dimensions.

To be continued...

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And others...