Wavelet analysis on the arrival directions of ultra high energy cosmic rays

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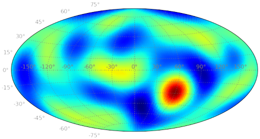

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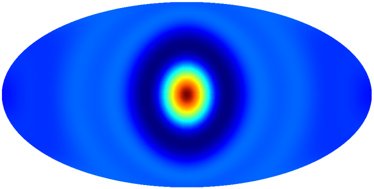
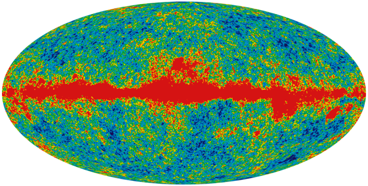
Outline

1. Motivation
2. Application of the wavelet
3. Anisotropy study
4. Summary and outlook



Anisotropy search in cosmic rays

Anisotropy in the cosmic microwave background.[1]



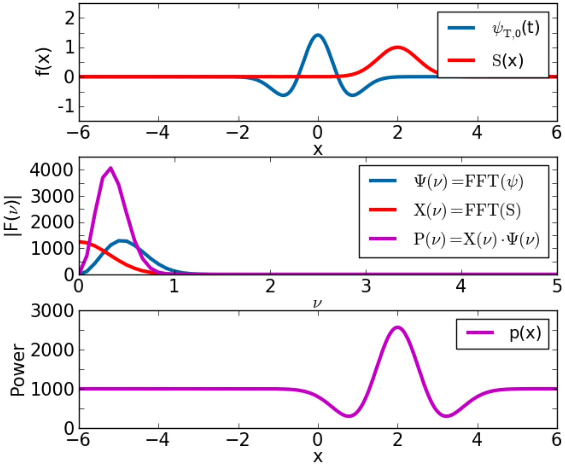
- What are the sources of ultra high energy cosmic rays?
- Charged particles are deflected in the extragalactic and the galactic magnetic field so they most likely arrive at the Earth isotropically distributed.
- UHECRs are less deflected and can point back to their source regions.
- $\rightarrow$ Goal of my thesis is to study the arrival directions of cosmic rays for point sources and large scale structures.

[1] http://lambda.gsfc.nasa.gov/

Motivation of a wavelet analysis

Wavelets are a short periodic functions to filter and to reconstruct local features.

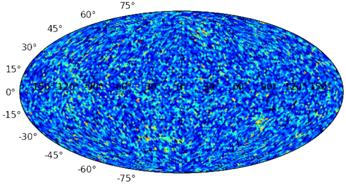
For example the 1-dim Mexican-Hat wavelet:



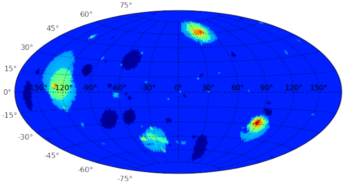
Motivation of spherical wavelet analysis

- Wavelet analysis is useful in the study of arrival directions of UHECRs.
- Spherical wavelet were used in the search for anisotropy of the cosmic microwave background.

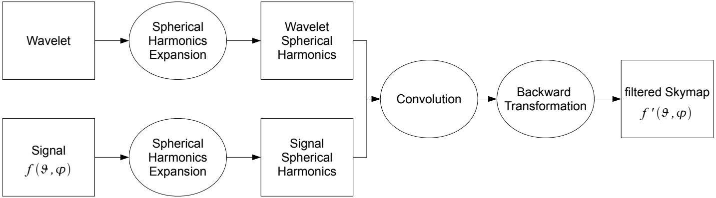
Monte Carlo UHECRs arrival directions with 4 point sources:



Spherical wavelet analysis result:



Wavelet analysis on the sphere



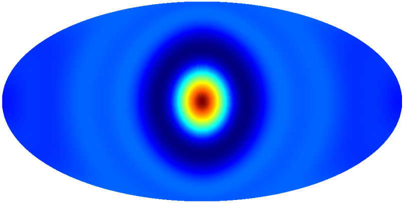
- Compute the spherical harmonics of the signal skymap and the spherical wavelet

$$f(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta, \varphi) \quad (1)$$

- Convolution of wavelet and signal by means of the spherical harmonics
- Backward transformation into spherical coordinates to get filtered skymap

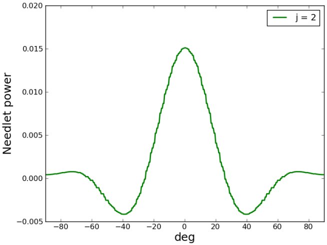
Introduction of the used wavelet 'needlet'

Needlet on the sphere:



Hammer-Aitoff-Projection

Needlet:



Lateral profile

Needlet features:

- good localization in spatial and frequency domain
- is a spherical wavelet introduced by the CMB community[2]
- can be easily implemented in Healpix, which is used for this analysis

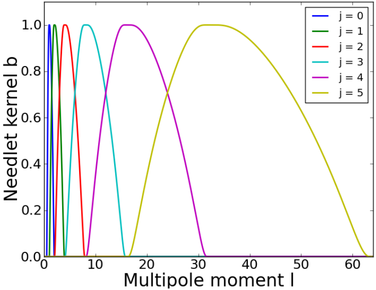
[2] Narcowich et al.2006 and Baldi et al. 2006

Needlet analysis

Pixel value in filtered skymap is defined as needlet power coefficient β_{jk}:

$$\beta_{jk} := \sqrt{\lambda} \sum_{l} b(l, B^{-j}) \sum_{m=-l}^{l} a_{lm} Y_{lm}(\xi_{jk}) \quad (2)$$

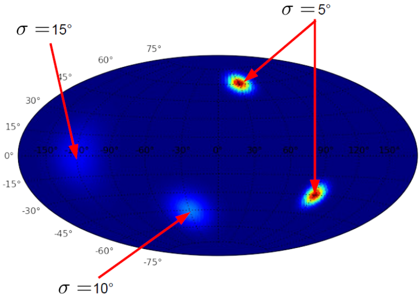
$b(l, B^{-j})$	= Needlet kernel function
B	= Needlet width (this analysis: B = 2)
j	= Needlet scale
l	= Multipole moment
a_{lm}	= Signal spherical harmonics
k	= Healpix pixel number
ξ_{jk}	= Healpix pixel center
λ	= Normalization factor



- Large j means small needlet size and therefore resolution of small structures
- Only small correlation of needlets with different scales

Generating MC test skymap for an ideal detector

Source density distribution:



MC skymap:

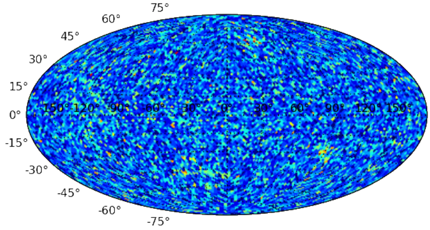
Source 1 with 100 events $\sigma = 5^\circ$

Source 2 with 100 events $\sigma = 5^\circ$

Source 3 with 200 events $\sigma = 10^\circ$

Source 4 with 300 events $\sigma = 15^\circ$

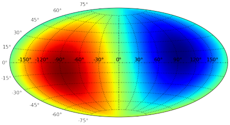
Test skymap with isotropic noise (30000 Events):



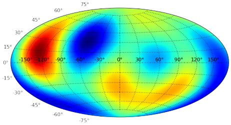
→ Now needlet analysis filters skymap to reconstruct the original source density distribution!

Needlet analysis of test skymap

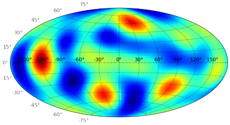
Filtered skymaps (power maps) for different needlet scales j:



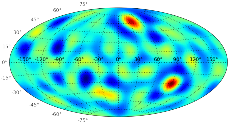
$j = 0$



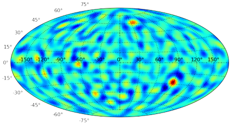
$j = 1$



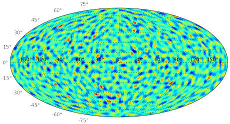
$j = 2$



$j = 3$



$j = 4$

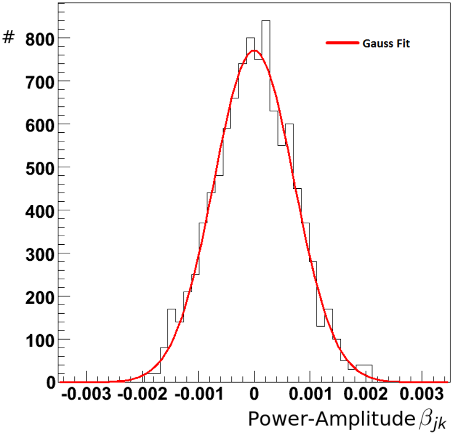


$j = 5$

- Structures are visible!
- $\rightarrow$ Mathematical method needed to find the significant entries in power maps

Threshold method

Goal: Reduce isotropic background and specify significant entries and regions
$\rightarrow$ Study power fluctuation (per pixel) from 10000 isotropic MC skymaps



β_{jk} = Power-Amplitude of scale j in Pixel k

$< \beta_{jk} >$ and σ_{jk} are determined with a Gauss-Fit

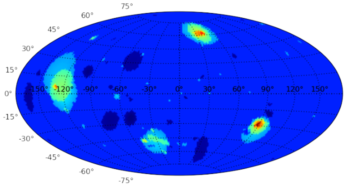
Significance per pixel:

$$S_{jk} = \frac{|\beta_{jk} - < \beta_{jk} > |}{\sigma_{jk}} \qquad (3)$$

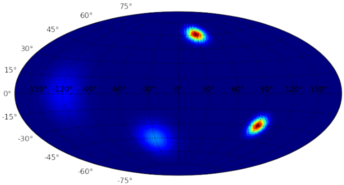
Threshold: Set all $S_{jk} < 3$ to zero

Comparison of resulting skymap with source density distribution

Sum of the 6 power maps for different needlet scales j after threshold:



Source Density Distribution:



All 4 sources are found!
$\rightarrow$ Now testing for global significance compared to isotropic skymap.

Global significance check

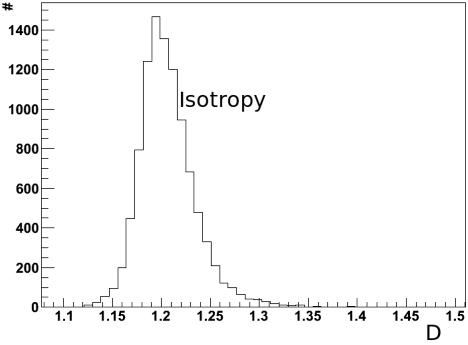
Significance value for whole skymap:

$$D := log \left(\frac{\sum_{k=1}^{N_{Pix}} |S_k|}{N_{nonzero}} \right) \qquad (4)$$

S_k = Power of pixel k of resulting skymap

N_{Pix} = Number of pixel

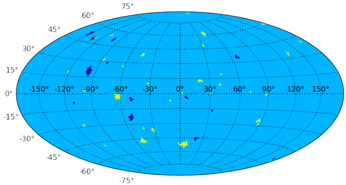
$N_{nonzero}$ = Number of pixel unequal zero

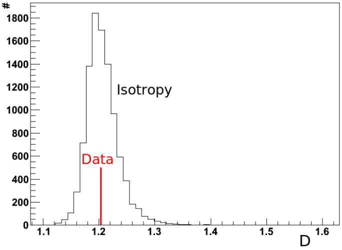


Distribution of 10000 isotropic MC skymaps

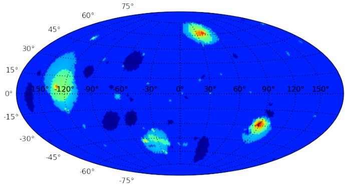
Applying of global significance check

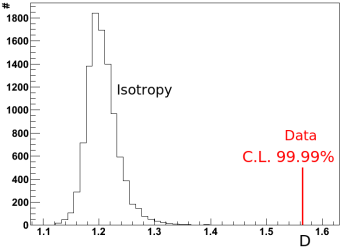
Isotropic MC skymap after threshold:





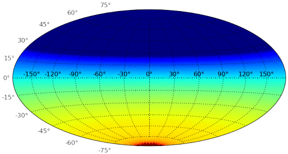
Skymap with signal after threshold:





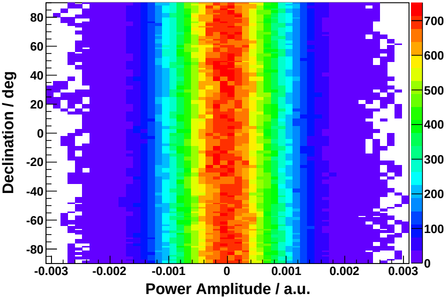
Resulting skymap can be discriminated from isotropy!

Due to the non-uniform exposure the power-estimator is declination dependent

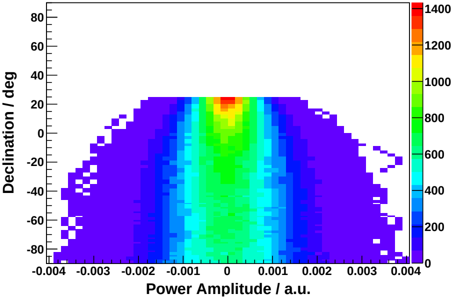


Declination dependency of β_{jk} (j = 2):

Fullsky:

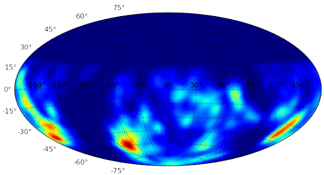


Considering Auger exposure:

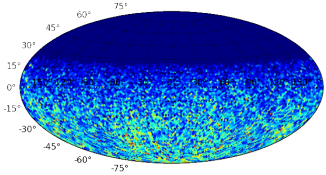


Applying Auger Exposure

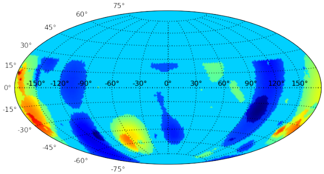
Source density distribution based on PSCz-Catalog:



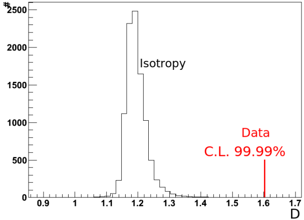
Signalmap:



Result after Threshold-Method:



Significance:



Method works also for an incomplete and non-uniform exposure!

Summary and outlook

Conclusion:

- Wavelet analysis can be used to filter event-based skymaps
- Threshold method can determine the original source density distribution
- Significance of the threshold results can be evaluated
- Method is working with every exposure
- Signal can be discriminated from isotropy!
- Applied method to Auger data (not shown here)

ToDo:

- Evaluate the detection limit
- Evaluate systematics effects and errors

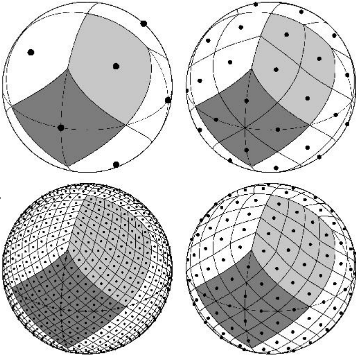
Backup Slides

Healpix

HEALPix:

- stands for Hierarchical Equal Area isoLatitude Pixelization of a sphere
- can easily transform a skymap into their spherical harmonics
- can apply weighted mask for incomplete sky coverage

Healpix Pixel Pattern[a]:



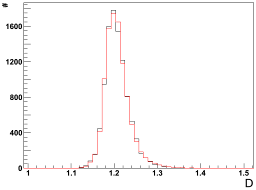
[a] http://healpix.jpl.nasa.gov

Anisotropy Detection Limit with MC

Monte-Carlo-Study with 10000 skymaps with different numbers of signal events

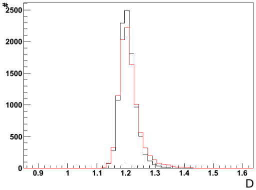
Source 1 = 0 events
Source 2 = 0 events
Source 3 = 0 events
Source 4 = 0 events

Sum 0 events



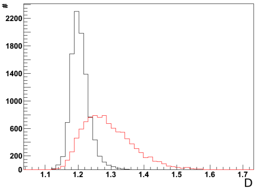
Source 1 = 30 events
Source 2 = 30 events
Source 3 = 60 events
Source 4 = 90 events

Sum 210 events



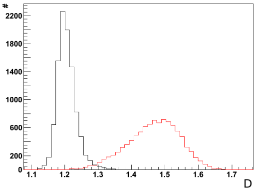
Source 1 = 60 events
Source 2 = 60 events
Source 3 = 120 events
Source 4 = 180 events

Sum 420 events



Source 1 = 100 events
Source 2 = 100 events
Source 3 = 200 events
Source 4 = 300 events

Sum 700 events



$\rightarrow$ Detection Limit needs to be evaluated