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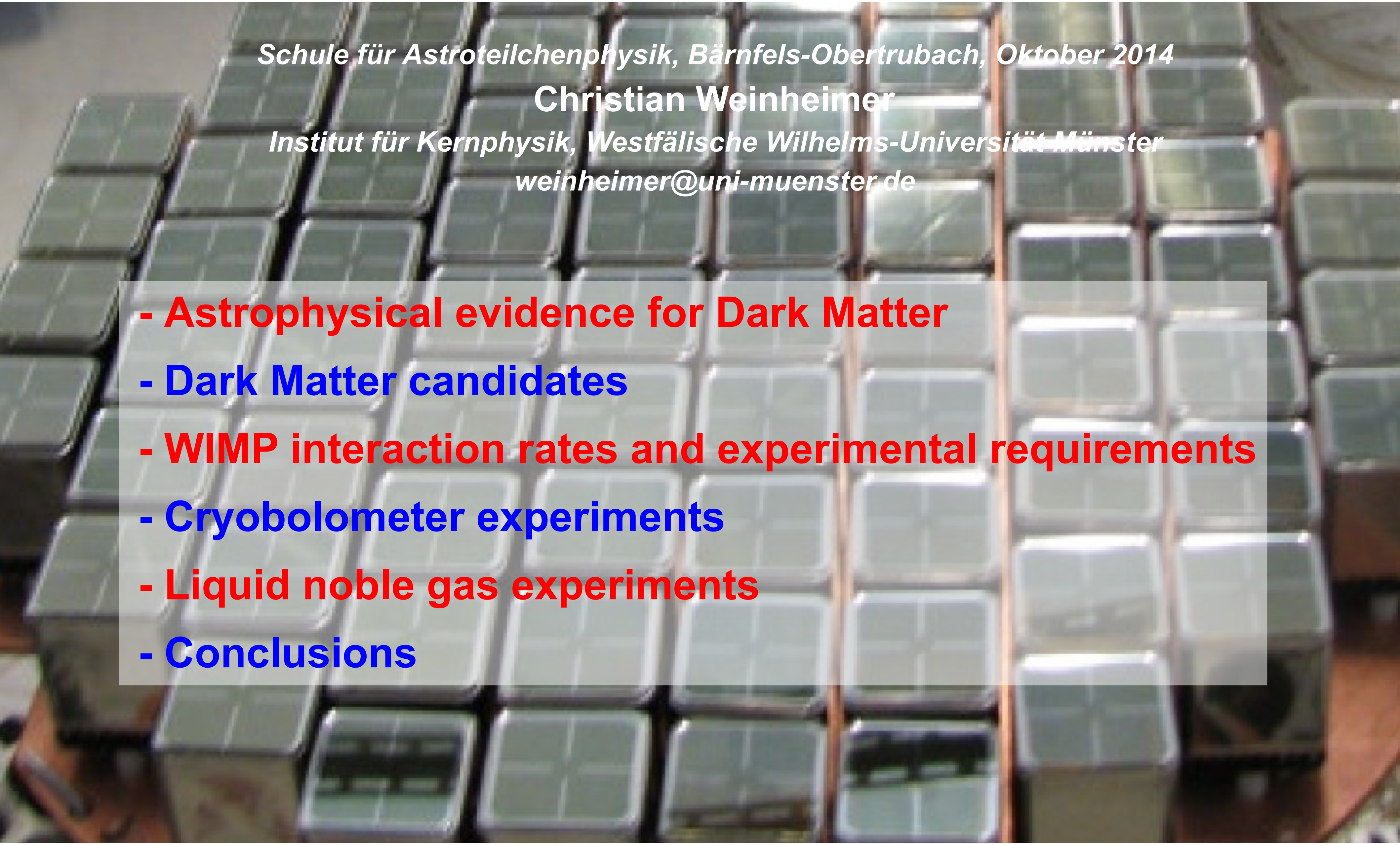
Direct Search for Dark Matter



Schule für Astroteilchenphysik, Bärnfels-Obertrubach, Oktober 2014

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- **Astrophysical evidence for Dark Matter**
- **Dark Matter candidates**
- **WIMP interaction rates and experimental requirements**
- **Cryobolometer experiments**
- **Liquid noble gas experiments**
- **Conclusions**

Summary of 1st lecture

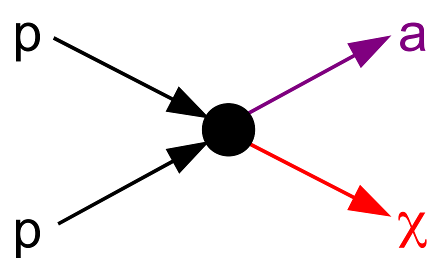
There is **compelling evidence** on all astrophysical scales
(rotation curve of galaxies, gravitational lensing, CMB, structure formation, ..)
for non-baryonic dark matter
5 times more than baryonic matter !

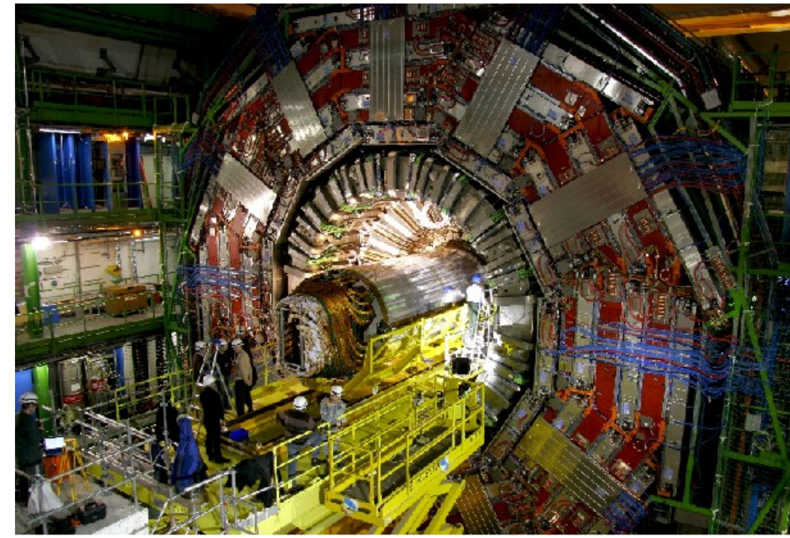
Possible candidates are many:
presently top candidates:
WIMPs (weakly interaction massive particle)
twice motivitated by WIMP miracle
very light **axions**
keV neutrinos

Experimental search for WIMPs I

a) At accelerators:

$p + p \to ..... \to ... + \tilde{a} + \tilde{\chi}$



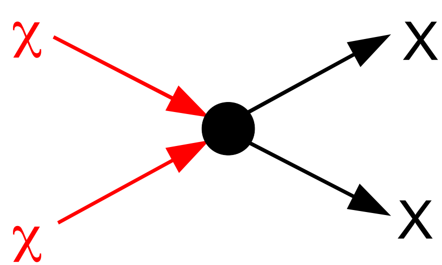


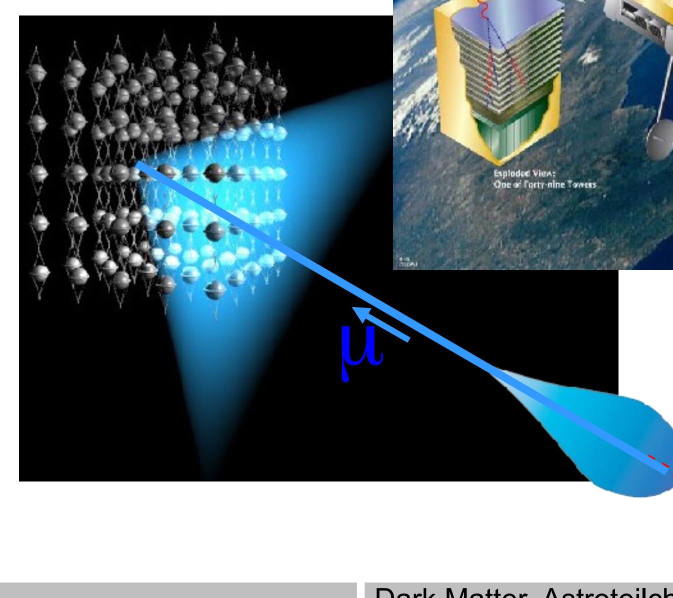
Indirect detection by missing mass+momentum
Not really a proof of WIMPs being the
Dark Matter of the universe

b) WIMP annihilation in the universe:

$\tilde{\chi} + \tilde{\chi} \to ..... \to ... + \nu + \bar{\nu}$

$..... \to ... + \gamma + \gamma$





Search for neutrinos or gammas from large
mass accumulations (center of galaxy, sun, ..)

Experimental search for WIMPs I

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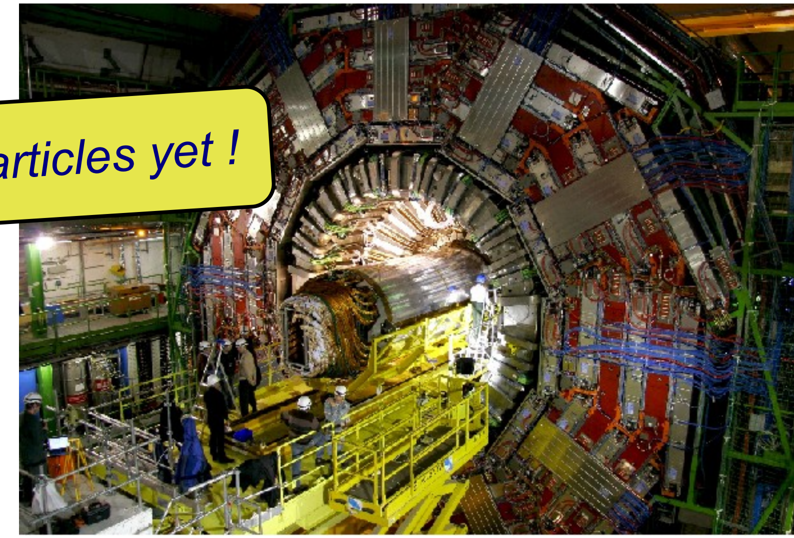
$p + p \rightarrow \ldots\ldots \rightarrow \ldots + \tilde{a} + \tilde{\chi}$

No indication for new particles yet !



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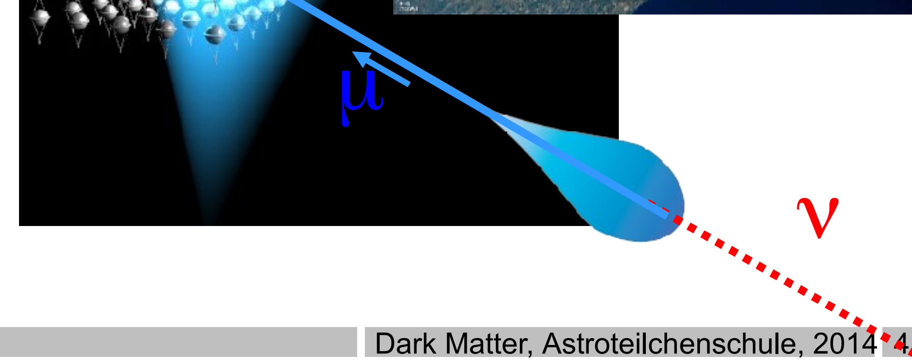

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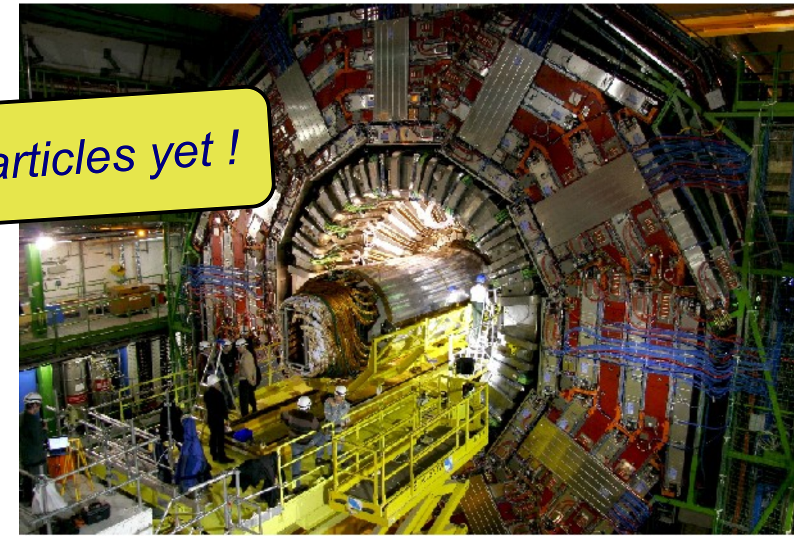

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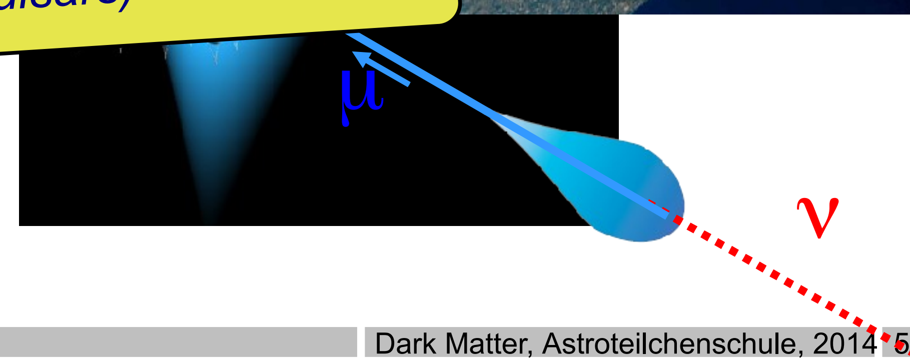
$$\tilde{\chi} + \tilde{\chi} \to ..... \to ... + \nu + \bar{\nu}$$

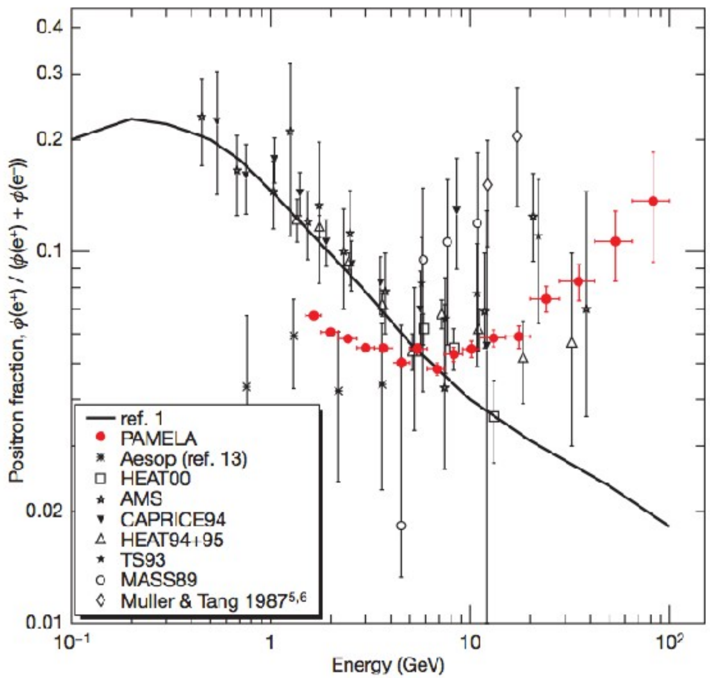
$$..... \to ... + \gamma + \gamma$$

Claimed lepton or positron excess can be also explained purely by astrophysics (e.g. pulsars)

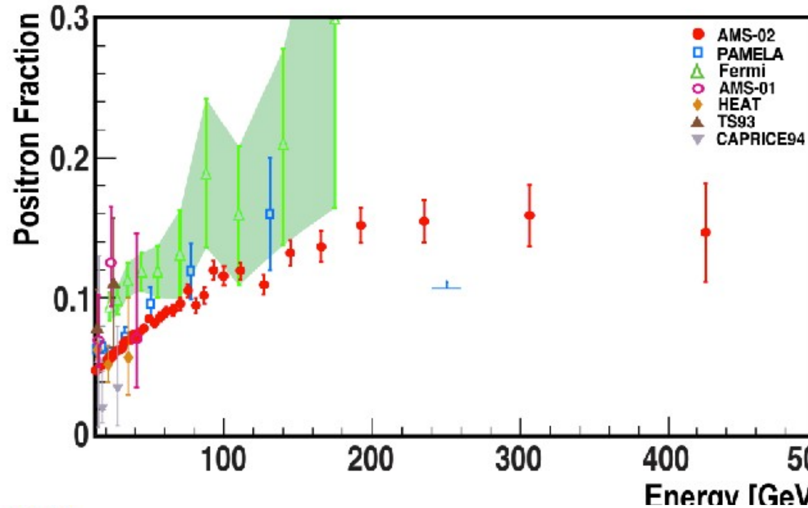


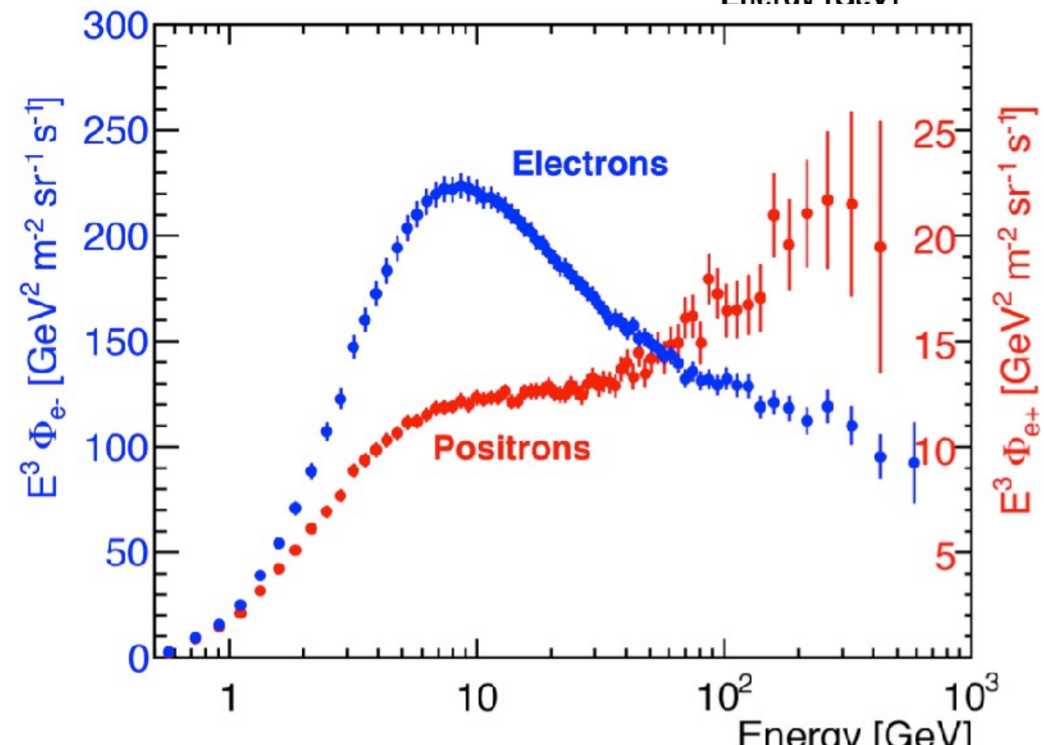
Search for neutrinos or gammas from large mass accumulations (center of galaxy, sun, ..)

Positron excess by PAMELA, latest results by AMS-II



O. Adriani et al., Nature (2009)

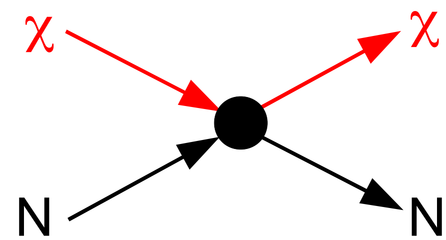


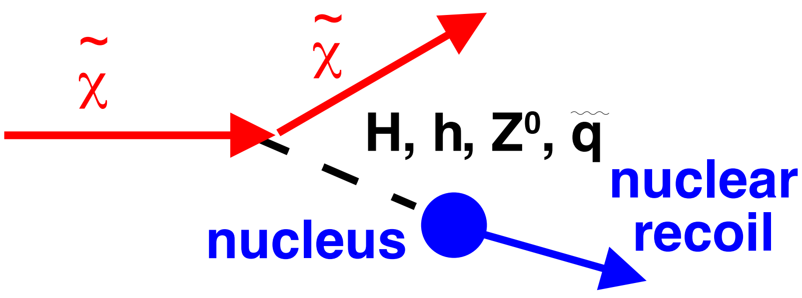


Excess is clearly confirmed but origin still unclear !

Experimental search for WIMPs II: Direct detection

c) Direct WIMP detection – search for nuclear recoil:





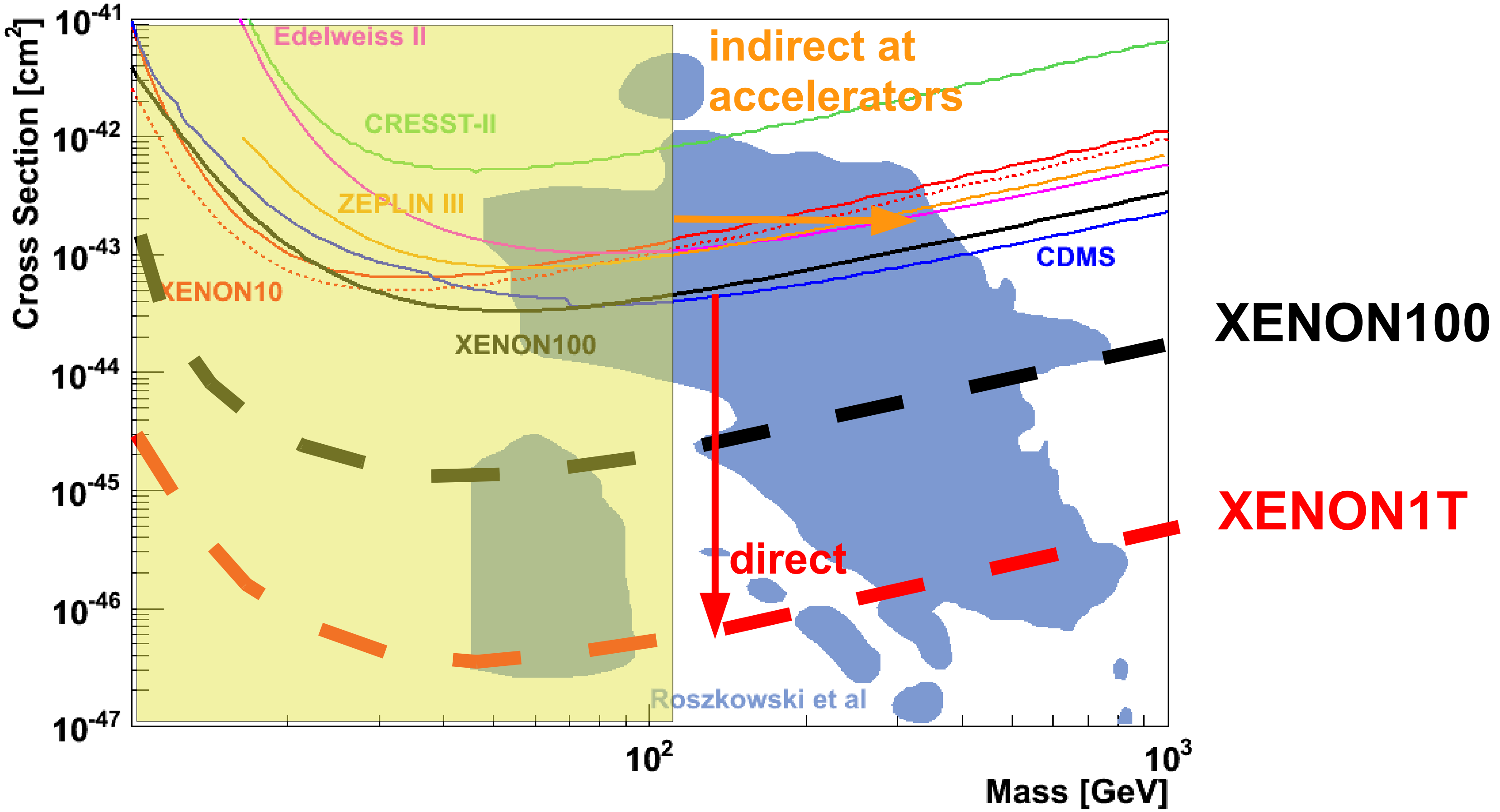
Signature: energy transfer to nucleus by "invisible particle" similar to neutrino NC interaction

Elastic coherent scattering on the nucleus mediated by H, h, Z, or $\tilde{q}$ exchange.

Effectively a scalar spin-independent (SI) or spin-dependent (SD) interaction

In principle even 6 couplings (+ 2 interference terms) to nuclear d.o.f. possible (arXiv:1308.6288)

Direct versus indirect searches



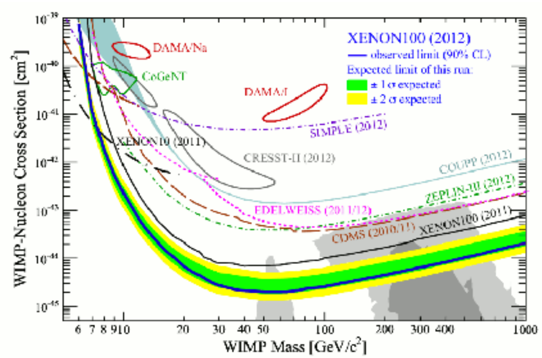
Nothing found yet, but region of SUSY is being attacked !

Theoretical WIMP cross section connected to experimental rate limit

1) elastic WIMP-quark interaction: $\sigma(\tilde{\chi} + q \rightarrow \tilde{\chi} + q)$

theory/models
e.g. χPT

2) elastic WIMP-nucleon interaction: $\sigma(\tilde{\chi} + N \rightarrow \tilde{\chi} + N)$



nuclear model
distribution of scattering objects:
Formfactors

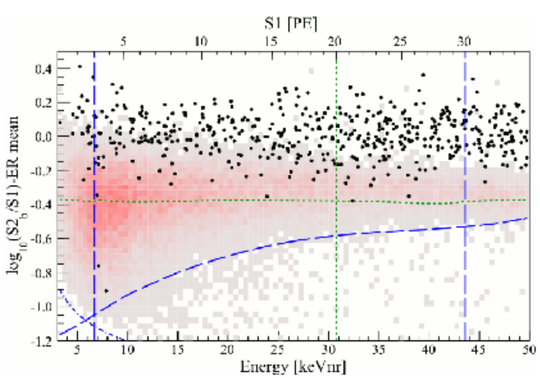
3) elastic WIMP-nucleus interaction: $\sigma(\tilde{\chi} + A \rightarrow \tilde{\chi} + A)$

astrophysics of WIMP halo:
ρ_0, $\bar{v}$, f(**v**)
particle physics: m_χ

4) theoretical recoil energy spectrum: $\frac{dR}{dE_r}(\tilde{\chi} + A \rightarrow \tilde{\chi} + A)$

material & detector properties

5) detected recoil spectrum: $\frac{d^2R}{dS_1\,dS_2}(\tilde{\chi} + A \rightarrow \tilde{\chi} + A)$



Reviews:

Jungmann, Kamionkowski and Griest, Phys. Rep. 267 (1996) 195

Levin and Smith, Astroparticle Phys. 6 (1996) 87

Notations

$$m_{\tilde{\chi}} \quad : \quad \text{mass of the WIMP}$$

$$m_{\mathrm{N}} \quad : \quad \text{mass of the nucleon}$$

$$m_{\mathrm{A}} \quad : \quad \text{mass of the nucleus}$$

$$\mu_{\mathrm{r}} = \frac{m_{\tilde{\chi}} \cdot m_{\mathrm{A}}}{m_{\tilde{\chi}} + m_{\mathrm{A}}} \quad : \quad \text{reduced mass}$$

$$E_{\mathrm{r}} \quad : \quad \text{recoil energy}$$

Relativitistic kinematics:

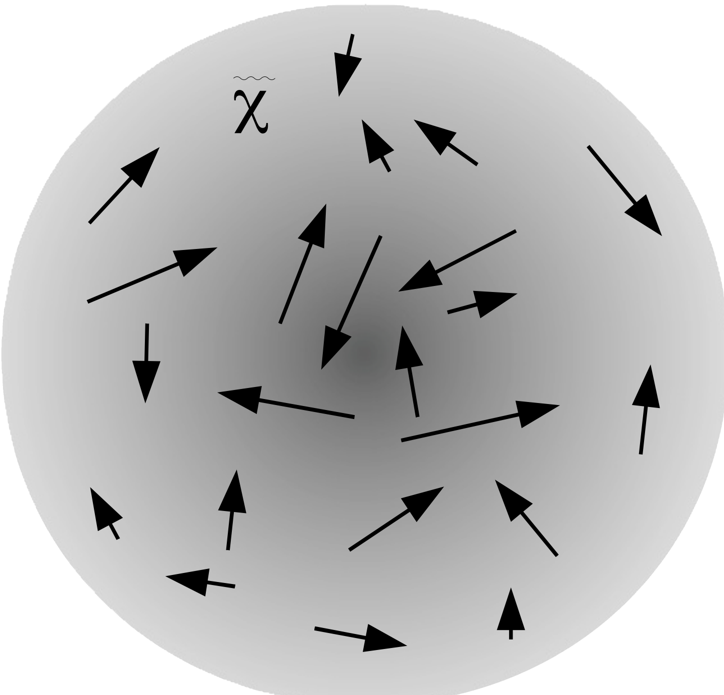
$$\mathbf{q}^2 = q_\mu q^\mu \quad \text{and} \quad Q^2 = -\mathbf{q}^2$$

But here non-relativitistic case ($m_{\tilde{\chi}} \gg E_{\mathrm{r}}$ and $m_{\tilde{\chi}}{}^2 \gg |\mathbf{q^2}|$):

$$q^2 := \vec{q}^{\,2}$$

$$\text{often used :} \quad Q = E_{\mathrm{r}}$$

WIMP velocity distribution



Assume a local 3-dim. Maxwellian WIMP velocity distribution:

$$f(\vec{v})\ d^3v = \frac{1}{\pi^{3/2}\ v_0^3} \cdot e^{-v^2/v_0^2}\ d^3v \quad \text{with} \quad v_0 = 220\ \text{km/s}$$

$$\text{Remark}: \quad \sigma_v = \frac{1}{\sqrt{2}} \cdot v_0 \quad \Rightarrow \quad \int\int\int f(\vec{v})\ d^3v = 1$$

Turn this into a 1-dim. velocity distribution by assuming radial symmetry:

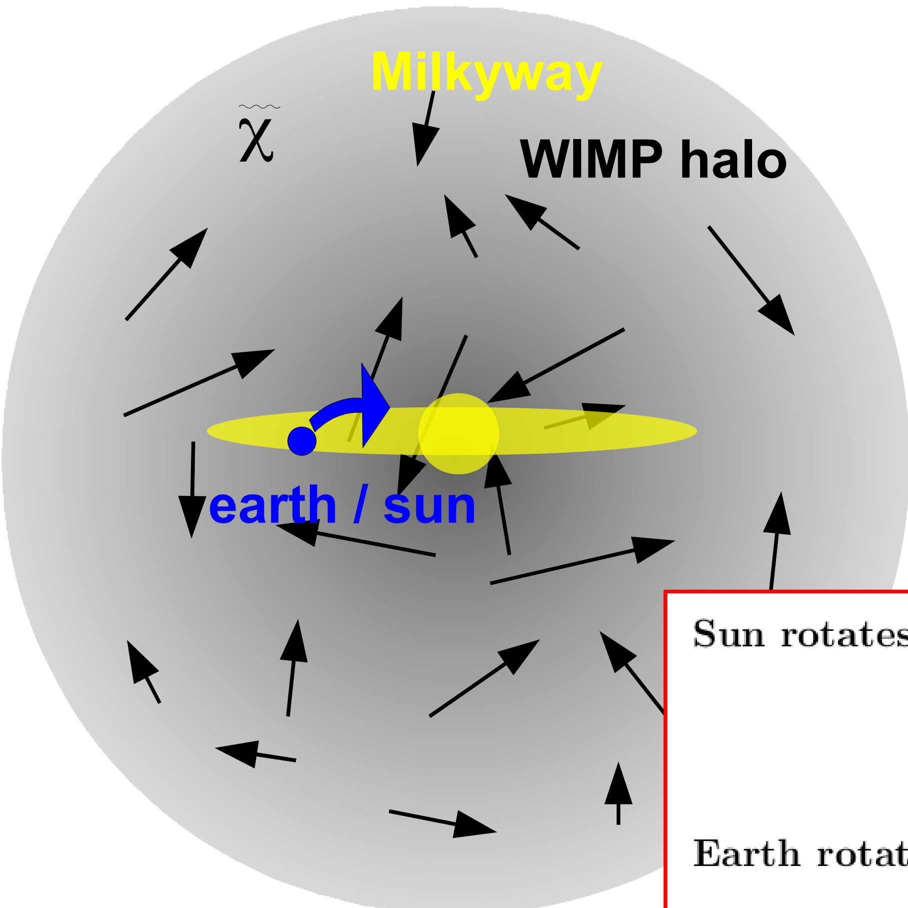
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\vec{v})\ d^3v = 4\pi \int_0^\infty \frac{v^2 \cdot e^{-v^2/v_0^2}}{\pi^{3/2}\ v_0^3}\ dv = \int_0^\infty \underbrace{\frac{4v^2 \cdot e^{-v^2/v_0^2}}{\sqrt{\pi}\ v_0^3}}_{:=f_1(v)} dv$$

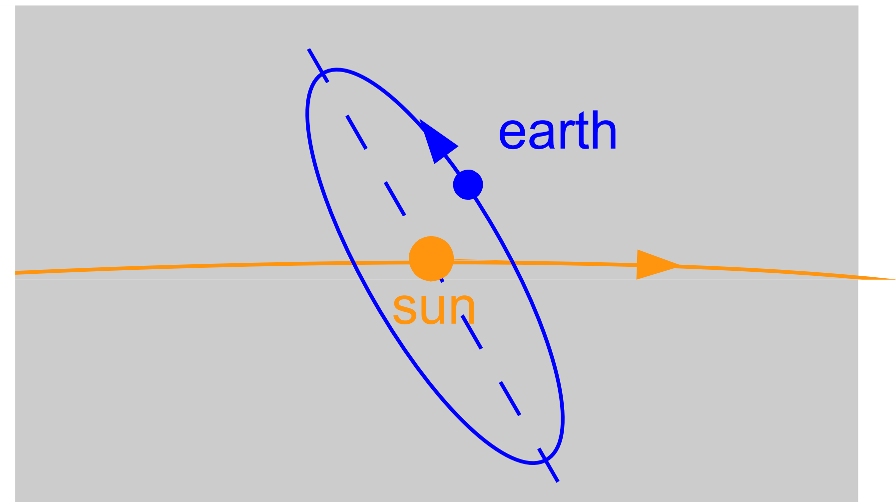
Velocity dispersion $\bar{v}$

$$\bar{v}^2 = \langle v^2 \rangle = \int_0^\infty v^2\ f_1(v)\ dv = \frac{3}{2}\ v_0^2$$

$$\Rightarrow \quad \bar{v} = \sqrt{\frac{3}{2}}\ v_0 = 270\ \frac{\text{km}}{\text{s}}$$

Astrophysics





Sun rotates around center of Milkyway with velocity

$$v_\odot = 230\ \frac{\text{km}}{\text{s}}$$

Earth rotates around sun with velocity

$$v_e(t) = v_e \cdot \cos(\omega t + \phi) \quad \text{with} \quad v_e = 30\ \frac{\text{km}}{\text{s}} \quad \text{and} \quad \frac{2\pi}{\omega} = 1\ \text{yr}$$

Velocity of earth w.r.t. center of Milkyway

$$\vec{v}_E(t) = \vec{v}_\odot + \vec{v}_e(t) \quad \text{with} \quad \angle(\vec{v}_\odot, \vec{v}_e) = 60^\circ$$

Kinematic relations

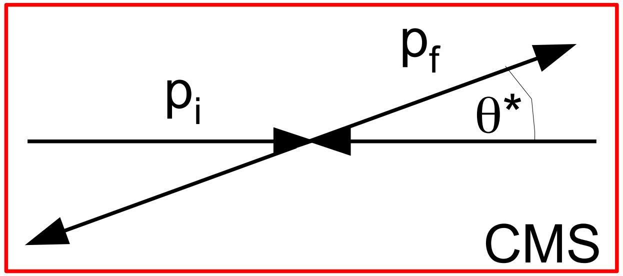
Describe elastic scattering in the center of mass system CMS with relative velocity v.

The center of mass moving with a velocity v_s is defined such, that the two momenta in CMS are equal (assume nucleus at rest in lab system)

$$m_{\tilde{\chi}} \cdot (v - v_s) = p = m_A \cdot v_s$$

$$\Rightarrow v_s = v \cdot \frac{m_{\tilde{\chi}}}{m_{\tilde{\chi}} + m_A}$$

$$\Rightarrow \quad p = m_A \cdot v_s = \mu_r \cdot v \quad \text{momentum in CMS}$$



Momentum transfer

$$\vec{q} = \vec{p}_i - \vec{p}_f \quad \text{with} \quad |p_i| = |p_f| = p = \mu_r \cdot v \quad \text{(elastic scattering)}$$

CMS scattering angle θ^*

$$\frac{q}{2} = p \cdot \sin\frac{\theta^*}{2}$$

$$\Rightarrow \quad q^2 = 4 \cdot p^2 \cdot \sin^2\frac{\theta^*}{2} = 2 \cdot p^2 \cdot (1 - \cos\theta^*) = 2 \cdot {\mu_r}^2 \cdot v^2 \cdot (1 - \cos\theta^*)$$

Recoil energy

$$E_r = \frac{q^2}{2 \cdot m_A} = \frac{{\mu_r}^2 \cdot v^2}{m_A} \cdot (1 - \cos\theta^*) \quad \Rightarrow \quad 0 \le E_r \le \frac{2 \cdot {\mu_r}^2 \cdot v^2}{m_A}$$

$$\frac{dq^2}{dE_r} = 2 \cdot m_A$$

Nuclear recoil energy E_r

in 0^{th} order:
assume s-wave scattering

Elastic scattering of a scalar (spin-independent) interaction:

$\Rightarrow$ angular distribution is uniformly distributed in the variable $\cos\theta^*$ within interval $[-1, 1]$

$\Rightarrow$ for a given WIMP velocity v the recoil energy E_r is uniformly distributed in interval $[0, \frac{2\cdot\mu_\mathrm{r}^2\cdot v^2}{m_A}]$

with $E_{\mathrm{r,\ max}} = \frac{2\cdot\mu_\mathrm{r}^2\cdot v^2}{m_A}$

Just to get a first estimate on the size of the recoil energy

Let us assume $m_{\tilde{\chi}} = 100\ \mathrm{GeV} = m_\mathrm{A} \quad (\Rightarrow \mu_\mathrm{r} = 50\ \mathrm{GeV})$

and $\theta^* = 90°$ and $v = v_0 \approx 0.7\cdot 10^{-3}$

$$\Rightarrow \quad E_\mathrm{r} = \frac{(50\ \mathrm{GeV})^2\cdot(0.7\cdot 10^{-3})^2}{100\ \mathrm{GeV}}\cdot(1-\cos 90°) = 25\ \mathrm{GeV}\cdot 0.5\cdot 10^{-6} = 12.5\ \mathrm{keV}$$

Scattering probability and cross section

Basic relation between cross section σ and interaction probabilty P and incoming flux density j

$$P = j \cdot \sigma \quad \text{with} \quad \vec{j} = \underbrace{n}_{\text{incoming particle density}} \cdot \underbrace{\vec{v}}_{\text{incoming particle velocity}}$$

With $N_{\rm T}$ target objects

$$\boxed{\dot{N}_{\rm T} = j \cdot \sigma \cdot N_{\rm T} = n \cdot v \cdot \sigma \cdot N_{\rm T}}$$
$$= n \cdot v \cdot \sigma \cdot \frac{M_{\rm T}}{m_{\rm A}} \quad \text{with} \quad M_{\rm T} \text{ being the total target mass, e.g. 34 kg of fid. vol.}$$

Remark: This relation is only valid for non-opaque targets, i.e. dimensions $\ll$ mean free path, which is usually fulfilled in weak interactions

The incoming WIMP density is given by the local (dark) matter density and the WIMP mass

$$n = n_{\tilde{\chi}} = \frac{\rho_0}{m_{\tilde{\chi}}}$$
$$\Rightarrow j \propto n \propto \frac{1}{m_{\tilde{\chi}}}$$

Recoil spectrum

Usually we are interested in the interaction reate per mass unit R

$$R = \frac{\dot{N}_T}{M_T} = n \cdot v \cdot \sigma \cdot \frac{M_T}{m_A} \cdot \frac{1}{M_T} = \frac{n \cdot v \cdot \sigma}{m_A} = \frac{\rho_0 \cdot v \cdot \sigma}{m_{\tilde{\chi}} \cdot m_A}$$

The WIMP velocity v is not fixed but follows a velocity distribution.

We first neglect the velocity of the earth $\vec{v}_E(t)$ and assume a radial symmetric velocity distribution $f_1(v)$ over which we have to integrate

$$R = \int dR = \int \frac{\rho_0 \cdot v \cdot \sigma \cdot f_1(v)}{m_{\tilde{\chi}} \cdot m_A} \, dv$$

We assume that the scalar coherent elastic cross section does not depend on the velocity of the WIMP ($\sigma = \sigma_0$) except for loss of coherence for higher energies, which we will consider later by introducing a form factor.

Then we consider a differential rate dR for a defined velocity in the range $[v, v + dv]$

$$\frac{dR}{dv} = \frac{\rho_0 \cdot v \cdot \sigma \cdot f_1(v)}{m_{\tilde{\chi}} \cdot m_A}$$

We are interested in the differential rate also in terms of the recoil energy E_r. Since for a giving velocity a scalar elastic interaction results in a uniform distribution of the recoil energy E_r in the interval $[0, E_{r,\ max}]$ we just devide $\frac{dR}{dv}$ by $E_{r,\ max} = \frac{2 \cdot \mu_r^2 \cdot v^2}{m_A}$

$$\frac{d^2R}{dv\ dE_r} = \frac{\rho_0 \cdot v \cdot \sigma \cdot f_1(v)}{m_{\tilde{\chi}} \cdot m_A \cdot E_{r,\ max}} = \frac{\rho_0 \cdot \sigma \cdot f_1(v)}{2 \cdot m_{\tilde{\chi}} \cdot v \cdot {\mu_r}^2}$$

Integrating over the velocity distribution yields

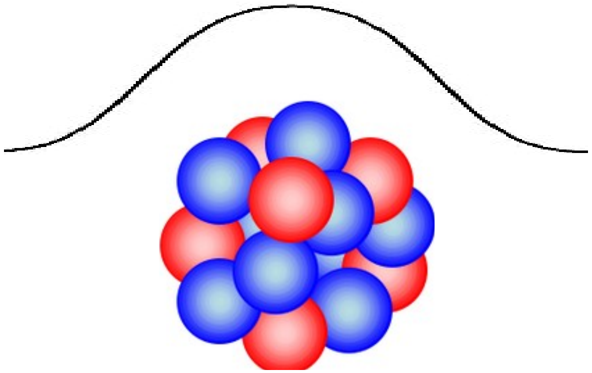
$$\frac{dR}{dE_r} = \frac{\rho_0 \cdot \sigma}{2 \cdot m_{\tilde{\chi}} \cdot {\mu_r}^2} \cdot \int \frac{1}{v} \cdot f_1(v)\ dv = \frac{\rho_0 \cdot \sigma}{2 \cdot m_{\tilde{\chi}} \cdot {\mu_r}^2} \cdot \langle \frac{1}{v} \rangle$$

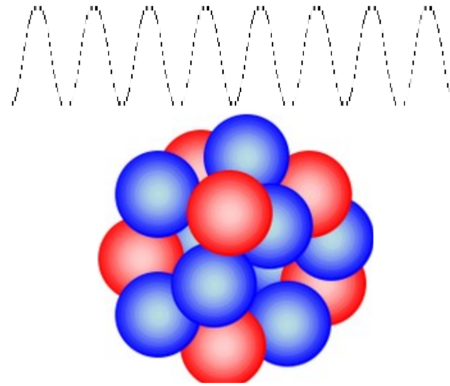
for simplification assume here $v_E = 0$

derivation can be repeated with $v_E \neq 0$

in 0th order: assume s-wave scattering

Increasing q^2: loss off coherence → Form factors





de Broglie wave length of the exchanged particle $\lambda = h / q$

Long wavelength:
nucleus seen as one object

Short wavelength:
sub-structure of nucleus is resolved

We now consider the loss of coherence for larger q^2 by introducing the Formfactor $F(q^2)$

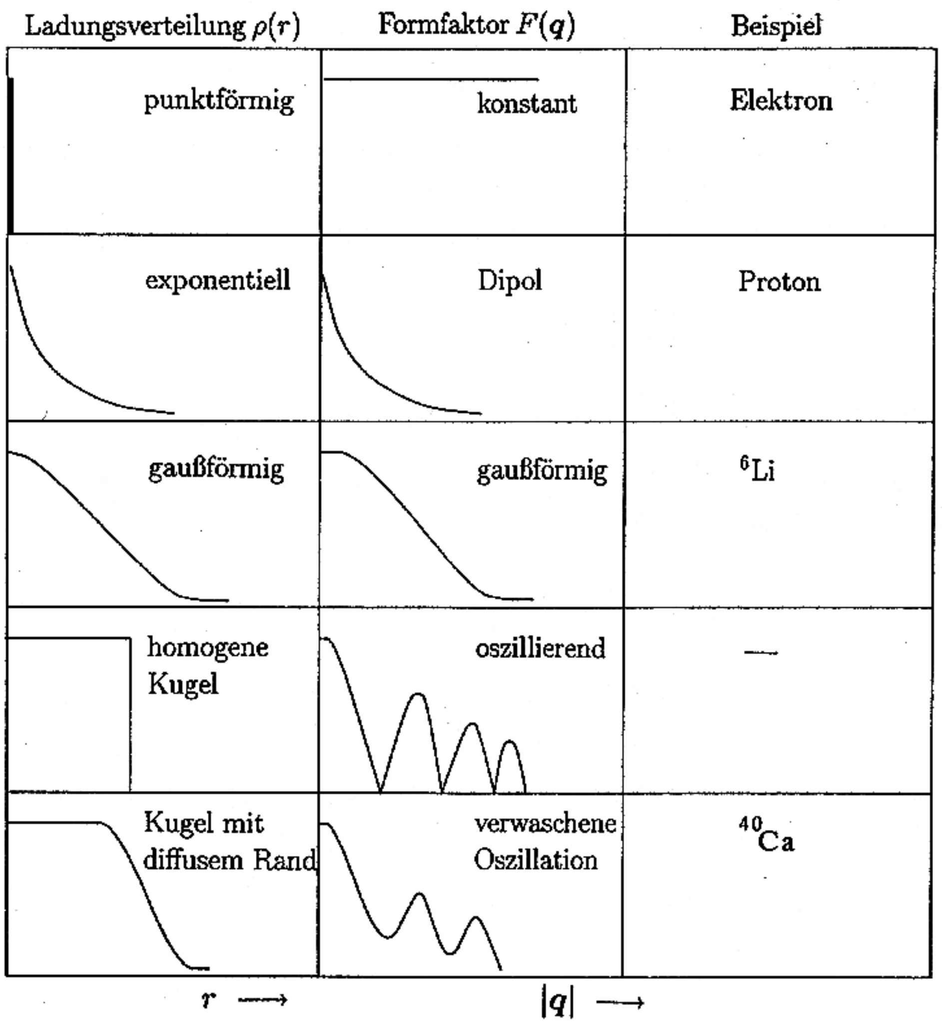
$$\sigma(q^2) = \sigma_0 \cdot F^2(q^2)$$

In low q^2 approximation the Formfactor is nothing than the Fouriertransform of the spatial distribution of the scattering objects,

e.g. the charge distribution for EM interaction
(which corresponds inside a nucleon to the weak charge (isospin) distribution for weak interactions from eN versus νN deep inelastic scattering)

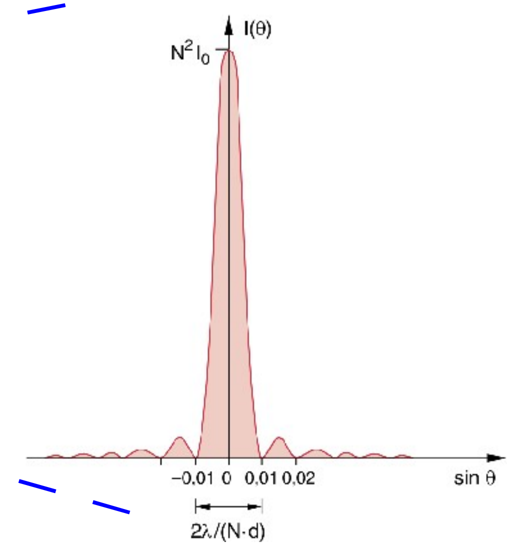
→ $F(q^2)$ has to be calculated by nuclear physics !

Form factors



Ladungsverteilung $f(r)$		Formfaktor $F(q^2)$	
Punkt	$\delta(r)/4\pi$	1	konstant
exponentiell	$(a^3/8\pi)\cdot\exp(-ar)$	$\left(1+q^2/a^2\hbar^2\right)^{-2}$	Dipol
Gauß	$(a^2/2\pi)^{3/2}\cdot\exp(-a^2r^2/2)$	$\exp\left(-q^2/2a^2\hbar^2\right)$	Gauß
homogene Kugel	$\begin{cases} C \text{ für } r \le R \\ 0 \text{ für } r > R \end{cases}$	$3\,\alpha^{-3}(\sin\alpha - \alpha\cos\alpha)$ mit $\alpha = \lvert q\rvert R/\hbar$	oszillierend

compare to interference pattern behind slit



pictures from Povh, Rith, Scholtz, Zetsche, *Teilchen und Kerne*, Springer

picture from Demtröder, *Experimentalphysik, Band 2*, Springer

Spin-independent and spin-dependent cross sections

WIMP-nucleus spin-independent (SI) and spin-dependent (SD) cross sections

$$\begin{aligned}
\sigma_{0,\mathrm{SI}} &= \frac{4}{\pi} \cdot {\mu_\mathrm{r}}^2 \cdot (Z \cdot f_\mathrm{p} + (A - Z) \cdot f_\mathrm{n})^2 \\
&= \frac{4}{\pi} \cdot {\mu_\mathrm{r}}^2 \cdot f^2 \cdot A^2 \quad \text{for} \quad f_\mathrm{p} = f_\mathrm{n} \\
&= \sigma_{\mathrm{N,SI}} \cdot \frac{{\mu_\mathrm{r}}^2}{m_\mathrm{N}^2} \cdot A^2 \\
\sigma_{\mathrm{N,SI}} &= \frac{4}{\pi} \cdot \left(\frac{m_\mathrm{N} \cdot m_{\tilde{\chi}}}{m_\mathrm{N} + m_{\tilde{\chi}}} \right)^2 \cdot f^2 = \frac{4}{\pi} \cdot m_\mathrm{N}^2 \cdot f^2 \quad \text{for} \quad m_{\tilde{\chi}} \gg m_\mathrm{N}
\end{aligned}$$

$$\sigma_{0,\mathrm{SD}} = \frac{32 \cdot {\mu_\mathrm{r}}^2}{\pi} \cdot \frac{J+1}{J} \cdot (a_p \cdot \langle S_p \rangle + a_n \cdot \langle S_n \rangle)^2 \cdot \frac{S(q)}{S(0)}$$

$$\text{with} \quad S(q) = a_0^2 \cdot S_{00}(q) + a_1^2 \cdot S_{11}(q) + a_0 \cdot a_1 \cdot S_{01}(q)$$

with $\langle S_p \rangle$ ($\langle S_n \rangle$) being the expectation values of the spin content of the protons (neutrons) in the nucleus and with the spin form factors S_{ij}

Expected recoil spectrum I

Evaluate $\langle 1/v \rangle$ The intergral does not range from 0 to ∞ because there is a lower velocity limit v_{min} to create a recoil of E_{r}

$$v_{\mathrm{min}} = \sqrt{\frac{E_{\mathrm{r}} \cdot m_{\mathrm{A}}}{2{\mu_{\mathrm{r}}}^2}}$$

There is also a maximum velocity $v_{\mathrm{max}} = v_{\mathrm{esc}}$ with 498 km/s $< v_{\mathrm{esc}} <$ 608 km/s (PDG2012), which is less important

With still the simplification $v_{\mathrm{E}} = 0$

$$\langle \frac{1}{v} \rangle = \int_{v_{\mathrm{min}}}^{v_{\mathrm{max}}} \frac{1}{v} \cdot f_1(v)\ dv = \int_{v_{\mathrm{min}}}^{v_{\mathrm{max}}} \frac{1}{v} \cdot \frac{4v^2 \cdot e^{-v^2/v_0^2}}{\sqrt{\pi}\ v_0^3}\ dv = \frac{4}{\sqrt{\pi}\ v_0^3} \cdot \int_{v_{\mathrm{min}}}^{v_{\mathrm{max}}} v \cdot \cdot e^{-v^2/v_0^2}\ dv$$

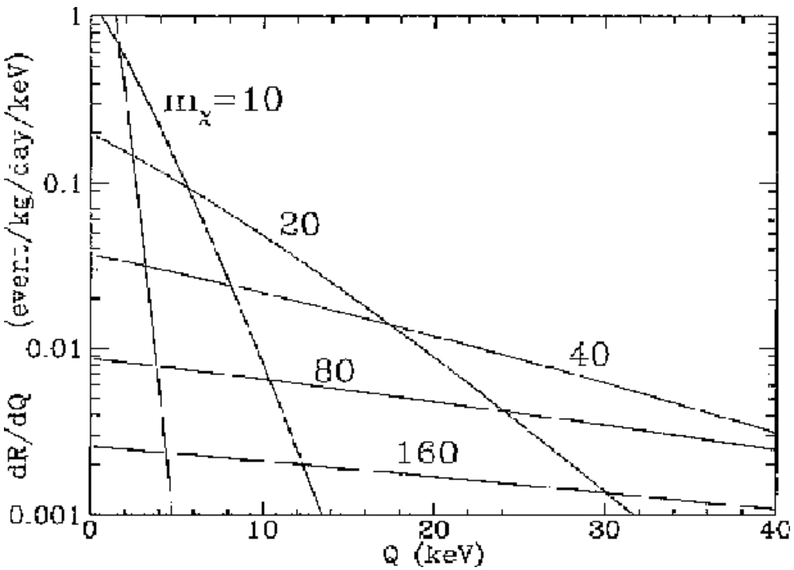
$$= \frac{4}{\sqrt{\pi}\ v_0^3} \cdot \left[-\frac{v_0^2}{2} \cdot e^{-v^2/v_0^2} \right]_{v_{\mathrm{min}}}^{v_{\mathrm{max}}} \approx \frac{4}{\sqrt{\pi}\ v_0^3} \cdot \left[-\frac{v_0^2}{2} \cdot e^{-v^2/v_0^2} \right]_{v_{\mathrm{min}}}^{\infty} = \frac{2}{\sqrt{\pi}\ v_0} \cdot e^{-v_{\mathrm{min}}^2/v_0^2}$$

$$= \frac{2}{\sqrt{\pi}\ v_0} \cdot e^{-\frac{E_{\mathrm{r}} \cdot m_{\mathrm{A}}}{2 \cdot \mu_{\mathrm{r}}^2 \cdot v_0^2}}$$

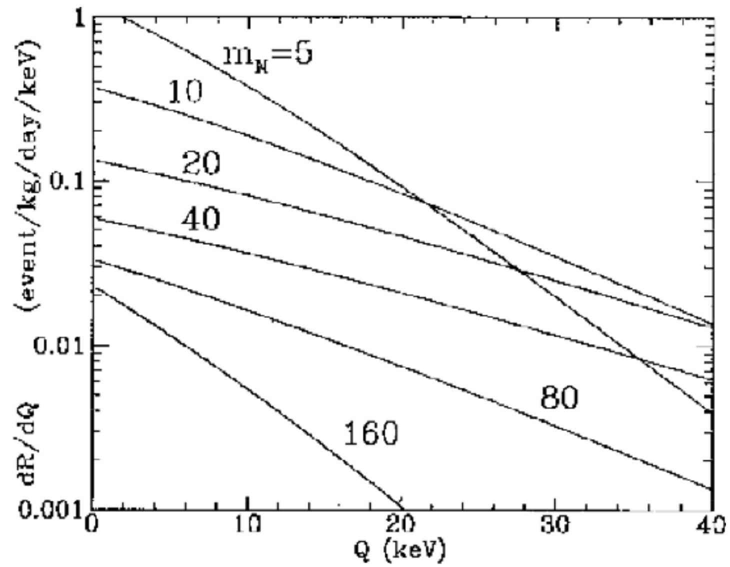
The **recoil spectrum** gets:

$$\frac{dR}{dE_{\mathrm{r}}} = \frac{\rho_0 \cdot \sigma_0 \cdot F^2(q^2)}{2 \cdot m_{\tilde{\chi}} \cdot {\mu_{\mathrm{r}}}^2} \cdot \langle \frac{1}{v} \rangle = \frac{\rho_0 \cdot \sigma_0 \cdot F^2(q^2)}{\sqrt{\pi} \cdot m_{\tilde{\chi}} \cdot {\mu_{\mathrm{r}}}^2 \cdot v_0} \cdot e^{-\frac{E_{\mathrm{r}} \cdot m_{\mathrm{A}}}{2 \cdot \mu_{\mathrm{r}}^2 \cdot v_0^2}}$$

Expected recoil spectra II



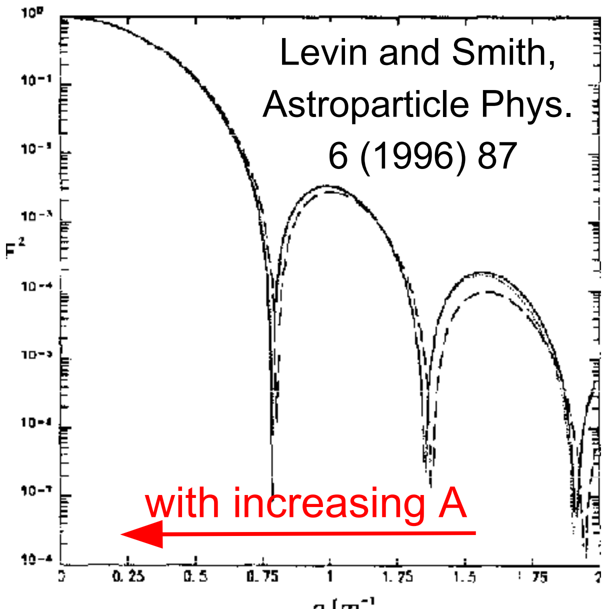
various m_χ, $m_A = m_{Ge}$, σ_0 = 4 10^{-36} cm^2



various m_A, m_χ = 40 GeV, σ_0 = 4 10^{-36} cm^2

but we can assume $\sigma_0 \sim A^2$

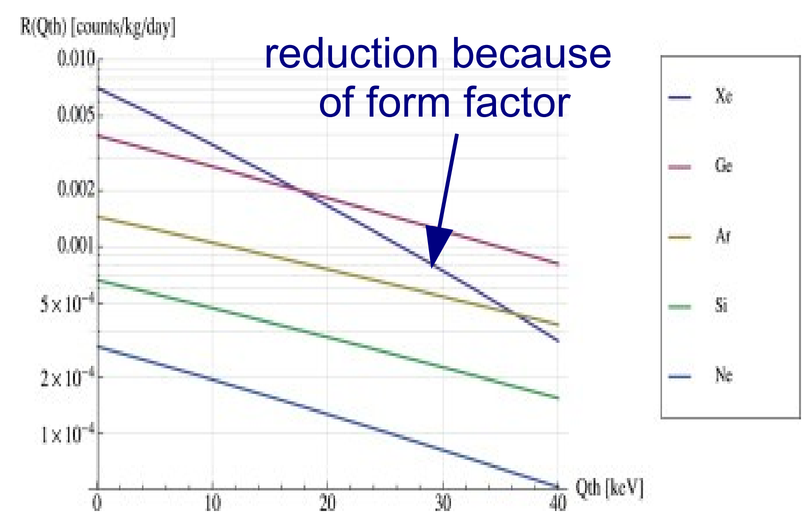
pictures from review of Jungmann, Kamionkowski and Griest, Phys. Rep. 267 (1996) 195



Levin and Smith, Astroparticle Phys. 6 (1996) 87

with increasing A

enhancement because of $\sigma_0 \sim A^2$



reduction because of form factor

picture from E. Figueroa-Feliciano, Prog. Part. Nucl. Phys. 66, 2011, 661

Summary of 2nd lecture

Expected nuclear recoil spectrum is a feature-less exponentially falling spectrum

$$\frac{dR}{dE_r} = \frac{\rho_0 \cdot \sigma_0 \cdot F^2(q^2)}{2 \cdot m_{\tilde{\chi}} \cdot \mu_r^{\,2}} \cdot \langle \frac{1}{v} \rangle = \frac{\rho_0 \cdot \sigma_0 \cdot F^2(q^2)}{\sqrt{\pi} \cdot m_{\tilde{\chi}} \cdot \mu_r^{\,2} \cdot v_0} \cdot e^{-\frac{E_r \cdot m_A}{2 \cdot \mu_r^2 \cdot v_0^2}}$$

Including earth movement around sun leads to a annual modulation of the rate and the spectrum

Require experimental threshold of O(10 keV) !

Larger nucleus mass is preferred, since scalar coherent interaction (SI) scales with A^2, but then smaller recoil energies !