



Lorentz Invariance Violation for High Cosmic Ray Propagation

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The Standard Model I

The SM is a **quantum field theory** with the following characteristics:

- Particle Content (fermions: Q,u,d, L, e, \dots scalars: H)
- Gauge Group: $SU(3)_c \times SU(2)_L \times U(1)_Y$
- Re-normalizable ($d=4$, $\hbar = c = 1$)
- **CPT invariant.** A system is said to have CPT Symmetry if the physics is unaffected by the combined transformations CPT
- **Lorentz Invariant** (space-time) . A physical system is said to have Lorentz Symmetry if the relevant laws of physics are unaffected by Lorentz transformations. → Lorentz scalars

What if... ?

The observation of Lorentz or CPT violation would be a sensitive signal for unconventional physics.

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Lorentz Symmetry $SO(3, 1)$

Lorentz Group:

- $SO(3)$
- Boost

Lorentz Violation:

- **By hand :**

Generic. (relatively easy to use, quick results; not fundamental, conservation laws?)

- **Spontaneous symmetry breaking:**

Preserve all the properties that we like from the SM.
Bjorken'63. Nambu'68.

By Hand

1. Add a non Lorentz invariant term to a generic free particle Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi + \epsilon \partial_i \phi \partial^i \phi \quad (+\epsilon \partial_0 \phi \partial^0 \phi)$$

2. Check the dispersion equation. (For p or E)

$$\implies E^2 - (1 + \epsilon)p^2 - m^2 = 0,$$

3. Since the LIV contributions are small, expand in Taylor form

$$E^2 - p^2 - m^2 = p^2 \left(\epsilon'(0) \cdot \frac{p}{M} + \epsilon'(0) \cdot \left(\frac{p}{M} \right)^2 + \dots \right)$$

4. Use very high energy limit. Then $A := \{E, p\}$. Keep only one term at time $n = 1, 2, \dots$

General LIV dispersion equation:

$$E^2 - p^2 = m^2 - \alpha_n A^{n+2}, \quad \alpha_n := \epsilon^{(n)} / M^n \approx 1/M^n$$

How these LIV correction change the Energy threshold for usual processes?

Pair Production: $\gamma_{VHE}\gamma_b \rightarrow e^+e^-$

$$\omega^2 - (1 + \alpha_n k^n)k^2 = 0, \quad E^2 - (1 + \alpha_n^+) p^2 = m^2$$

$$\frac{[\alpha_n - \alpha_n^+ K^{n+1} - \alpha_n^- (1 - K)^{n+1}]}{4\omega_b \frac{m_e^2}{4\omega_b K(1-K)}} \omega^{n+2} - \frac{1}{\frac{m_e^2}{4\omega_b K(1-K)}} \omega + 1 = 0$$

LI regime is recovered if

$$\begin{aligned} \alpha_n &= \alpha_n^\pm = 0 \\ K &= 1/2 \end{aligned} \Rightarrow$$

$$\boxed{\omega_{th}^{LI} = \frac{m_e^2}{\omega_b}}$$

Redefine:

$$x := \frac{4\omega_b K(1 - K)}{m_e^2} \omega := \frac{1}{\omega_{LI}} \omega$$

$$\Lambda_n := \frac{\omega_{LI}^{n+1}}{4\omega_b} \beta_n; \quad \beta_n := [\alpha_n - \alpha_{n+} K^{n+1} - \alpha_{n-} (1 - K)^{n+1}]$$

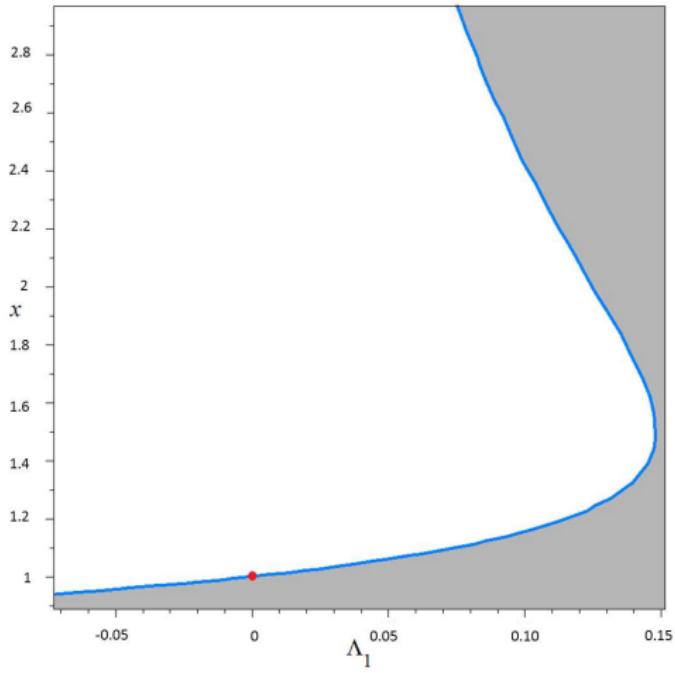
$$\implies \Lambda_n x^{n+2} - x + 1 = 0$$

$$\gamma VHE \gamma b \rightarrow e^+ e^- , \quad p_{CR} \gamma b \rightarrow p \pi^0 \implies \Lambda_n x^{n+2} - x + 1 = 0$$

Phenomenology for $n=1$

Three kinds of scenarios emerge:

- $\Lambda = 0$. LI is preserved. (red dot)
- $\Lambda < 0$. E_{th} moves backward from the standard E_{th}^{LI} .
- $\Lambda > 0$. E_{th} moves forward and a second E_{th} appears.

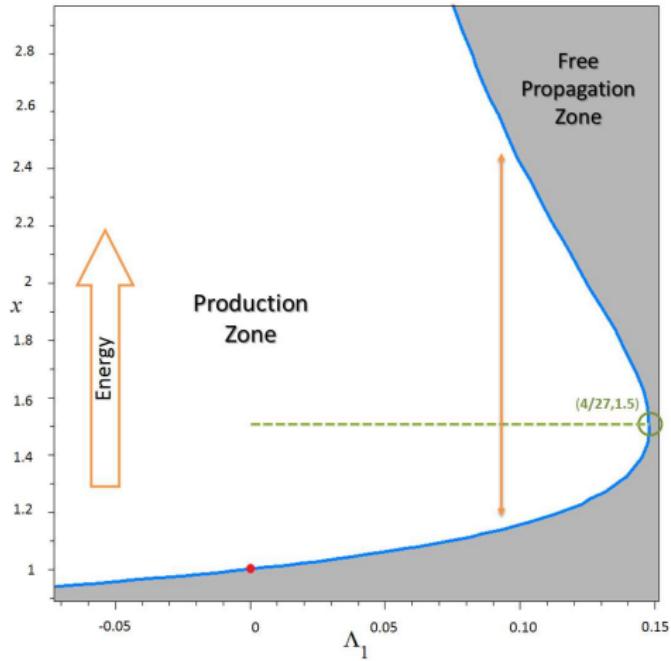


$$\gamma_{VHE} \gamma_b \rightarrow e^+ e^- , \quad p_{CR} \gamma_b \rightarrow p \pi^0 \quad \Rightarrow \quad \Lambda_n x^{n+2} - x + 1 = 0$$

Phenomenology for n=1

Three kinds of scenarios emerge:

- $\Lambda = 0$. LI is preserve. (red dot)
 - $\Lambda < 0$. E_{th} moves backward from the standard E_{th}^L .
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$\gamma VHE \gamma_b \rightarrow e^+ e^-$

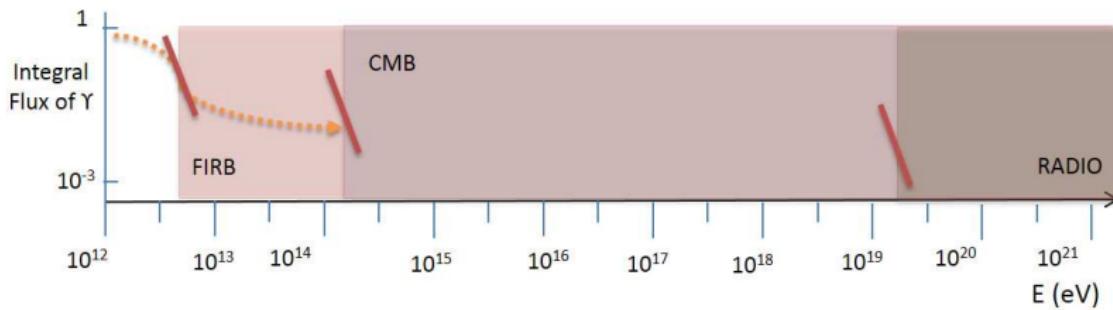
Energy threshold for several γ_b

- $\omega_{thFIRB}^{LI} \sim 3 \times 10^{13}$ eV
- $\omega_{thCMB}^{LI} \sim 5 \times 10^{14}$ eV
- $\omega_{th_{radio}}^{LI} \sim 6 \times 10^{19}$ eV

$\omega_{th's}^{LIV} = ?$

Generic example 1:

$$\Lambda_1 < 0$$

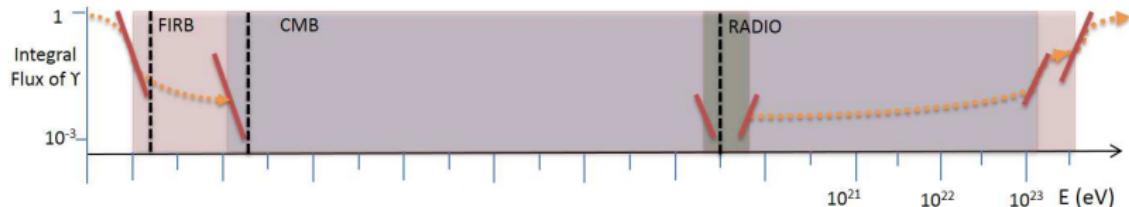


ω_{th}^{LIV} moves backward from the value of the standard ω_{th}^{LI} . And it happens for each pair-production region.

Generic example 2:

$$\Lambda_1 > 0$$

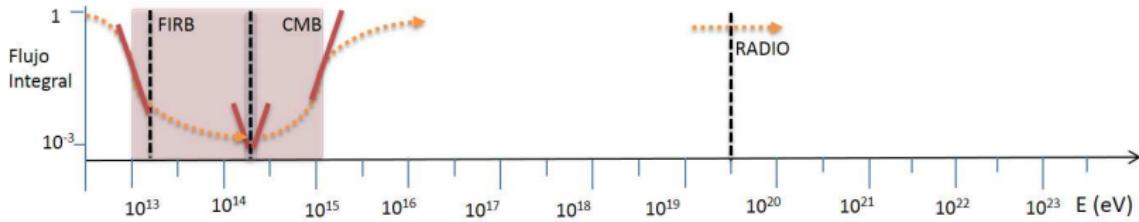
$$\beta_1 = 1.8 \times 10^{-48} eV^{-1}$$



Generic example 3:

$$\Lambda_1 > 0$$

$$\beta_1 = 1.1 \times 10^{-32} eV^{-1}$$



The γ_{HE} flux recovers at higher energies!

The background opacity region \longrightarrow '**opacity bands**' γ_{HE} . The width of the bands depends on the size of LIV coefficients.

There are several observational windows for the same model.

Unique signature of LIV.

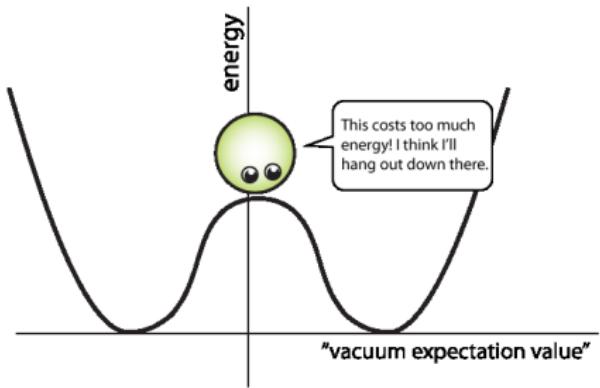
Spontaneously Symmetry Breaking

$$\mathcal{L} = \frac{1}{2}[(\partial\phi)^2 + \mu^2\phi^2] - \frac{\lambda}{4}(\phi^2)^2$$

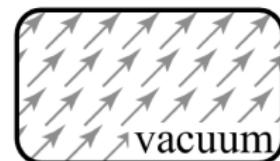
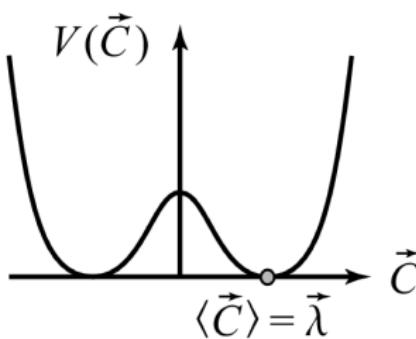
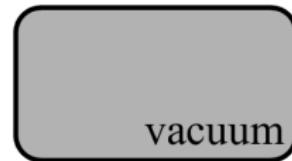
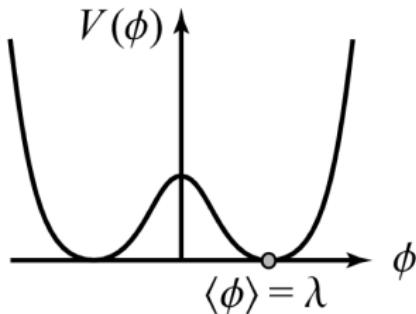
Two minima at $\phi = \pm v = \pm(\mu/\lambda)^{\frac{1}{2}}$.

We have to commit to one or the other of the two possibilities for the ground state and build perturbation theory around it.

We did not put symmetry breaking terms into the Lagrangian by hand but yet the reflection symmetry is broken.



The reflection symmetry is broken spontaneously!



SME: Pure-photon sector

1. Take de SM \mathcal{L} . You can do sector by sector.

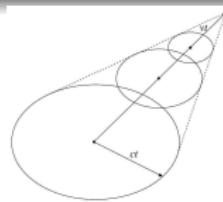
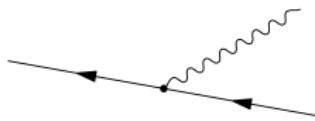
$$\mathcal{L}_{\text{photon}}^{\text{SM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (+\mathcal{L}_{\text{Dirac}})$$

2. Add the SME terms

$$\mathcal{L}_{\text{photon}}^{\text{CPT-even}} = -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}; \quad \mathcal{L}_{\text{photon}}^{\text{CPT-odd}} = +\frac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu}$$

3. Start the hunt

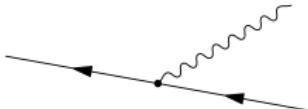
Vacuum Cherenkov Radiation



Brett, PRL 98, 041603,2007,

C. Kaufhold, F.R. Klinkhamer and M.
Schreck, 2008.

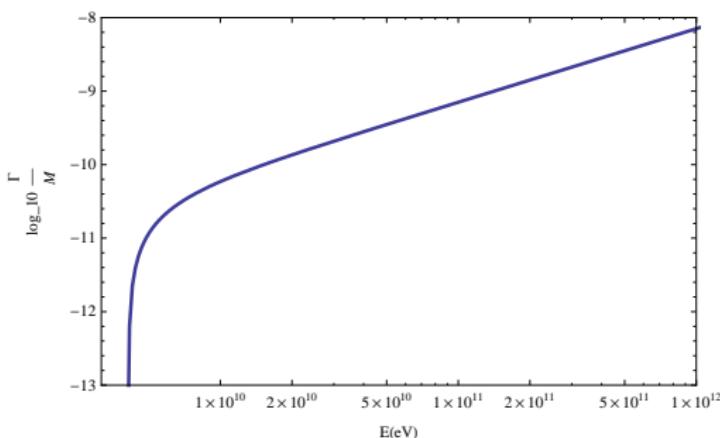
SSB: SME-Pure Photon Sector.



BH:

$$\omega^2 = k^2(1 + \alpha_n k^n)$$

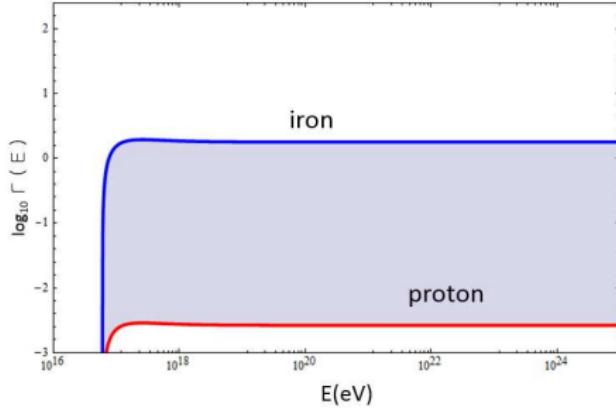
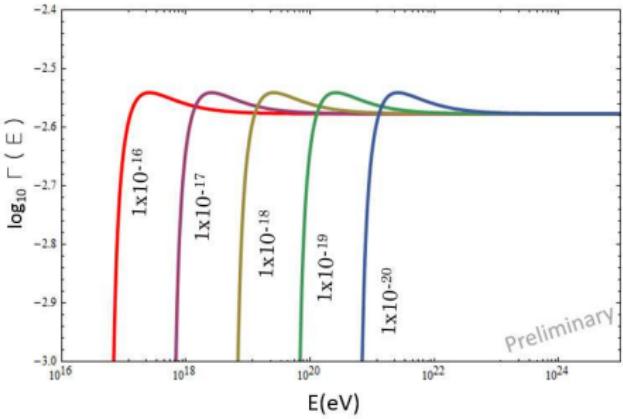
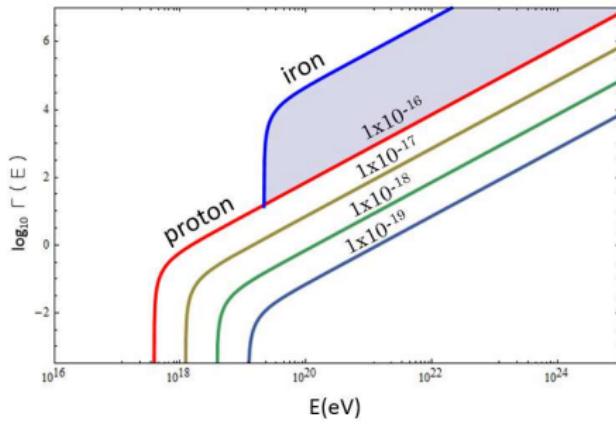
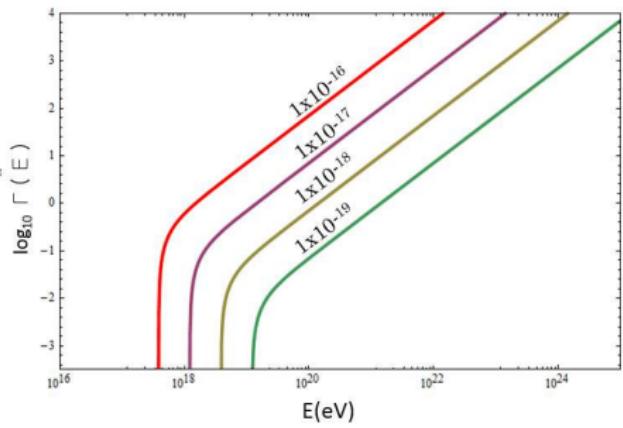
$$d\Gamma = \frac{S}{2} \frac{1}{E(q)} |\mathbf{M}|^2 \frac{d^3 p}{(2\pi)^3 2E_p(p)} \frac{d^3 k}{(2\pi)^3 2\omega(\mathbf{k})} (2\pi)^4 \delta^{(4)}(q - p - k)$$



Brett, PRL 98, 041603, 2007,

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Schreck, 2008.



While we would like to believe that the fundamental laws of Nature are symmetric, a completely symmetric world would be rather dull, and as a matter of fact, the real world is not perfectly symmetric. More precisely, we want the Lagrangian, but not the world described by the Lagrangian, to be symmetric. Zee.

Thanks! ☺

General Expression

$$E^2 - p^2 = m^2 + \alpha_n p^2 \left(\frac{p}{M}\right)^n$$

Particular Cases

Galaverni, Sigl, 2008; Maccione, Liberati, 2008; Jacobson, Liberati y Mattingly, 2003; Aloisio, Blasi, Ghia, Grillo, 2000

- $E^2 - p^2 = m^2 - \alpha_1 E p^2 - \alpha_2 E^3$ *Giovanni Amelino-Camelia, 2011*
 $E_T = E_1 + E_2 + \beta_1 \vec{p}_1 \cdot \vec{p}_2 + \beta_2 E_1 E_2;$
 $\vec{p}_T = \vec{p}_1 + \vec{p}_2 + \gamma_1 E_1 \vec{p}_2 + \gamma_2 E_2 \vec{p}_1;$ $\alpha_1 + \alpha_2 + \beta_1 + \beta_2 - \gamma_1 - \gamma_2 = 0;$
- **DSR 1** $E^2 - p^2 = m^2 - \alpha_1 E p^2$ $\beta_1, \beta_2, \gamma_1, \gamma_2 = 0;$
Amelino-Camelia & Ia., 2011
- $E^2 - p^2 = m^2 - \alpha_1 E p^2;$ *Giovanni Amelino-Camelia, 2011*
 $E_T = E_1 + E_2 + \beta_1 p_1 p_2$
 $\vec{p}_T = \vec{p}_1 + \vec{p}_2 + \gamma_1 E_1 \vec{p}_2 + \gamma_2 E_2 \vec{p}_1;$ $(\vec{p} = p \hat{e}_1);$
- $E^2 - p^2 = m^2 - \alpha_2 E^3$
...
- $E^2 = p^2 c_{MAV}^4 + m^2 c_{MAV}^4$ *Scully y Stecker, 2009*
- $m^2 = (1 + g(\alpha \frac{p^2}{E^2})) (E^2 - p^2)$ *Galaverni, Sigl, 2008*

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LI regime is recovered if

$$\begin{aligned} \alpha_n &= \alpha_n^\pm = 0 \\ K &= 1/2 \end{aligned} \Rightarrow$$

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Redefine:

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$$\Lambda_n := \frac{\omega_{LI}^{n+1}}{4\omega_b} \beta_n; \quad \beta_n := [\alpha_n - \alpha_{n+} K^{n+1} - \alpha_{n-} (1 - K)^{n+1}]$$

$$\implies \Lambda_n x^{n+2} - x + 1 = 0$$

Pion Production: $p_{CR}\gamma_b \rightarrow p\pi^0$

$$\omega^2 - (1 + \alpha_n k^n)k^2 = 0, \quad E^2 - (1 + \alpha_n^{p,p',\pi})p^2 = m^2$$

$$\frac{\left[\alpha_n'^p + \alpha_n^p(1-K)^{n+1} + \alpha_n^\pi K^{n+1}\right]}{\left[m_\pi^2 + \frac{K^2 m_p^2 + (1-K)^2 m_\pi^2}{(1-K)K}\right]} E_p^{n+2} - \frac{4\omega_b}{\left[m_\pi^2 + \frac{K^2 m_p^2 + (1-K)^2 m_\pi^2}{(1-K)K}\right]} E_p + 1 = 0$$

LI regime is recovered if $\alpha'^p = \alpha^p = \alpha^\pi = 0$
 $K = m_\pi/(m_\pi + m_p)$

$$\Rightarrow$$

$$E_p^{LI} = \frac{m_\pi(2m_p + m_\pi)}{4\omega_b}$$

Redefine:

$$x = \frac{4\omega_b}{m_\pi^2 + \frac{K^2 m_p^2 + (1-K)^2 m_\pi^2}{(1-K)K}} E_p := \frac{1}{E_{p_{th}}^{LI}} E_p;$$

$$\Lambda_n := \frac{(E_p^{LI})^{n+1}}{4\omega_b} \zeta_n; \quad \zeta_n := \left[\alpha_n'^p + \alpha_n^p(1-K)^{n+1} + \alpha_n^\pi K^{n+1} \right]$$

$$\implies \Lambda_n x^{n+2} - x + 1 = 0$$

Standard Model of Particles II

$$L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L, \quad R_A = (l_A)_R, \quad Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L, \quad U_A = (u_A)_R, \quad D_A = (d_A)_R,$$

A = 1, 2, 3 labels the flavor: $l_A = (e, \mu, \tau)$, $\nu_A = (\nu_e, \nu_\mu, \nu_\tau)$, $u_A = (u, c, t)$, $d_A = (d, s, b)$.

The Higgs doublet: $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ r_\phi \end{pmatrix}$; the Gauge Fields: G_μ, W_μ and $G_{\mu\nu}$

$$\mathcal{L}_{lepton} = \frac{1}{2} i \bar{L}_A \gamma^\mu \overleftrightarrow{D}_\mu L_A + \frac{1}{2} i \bar{R}_A \gamma^\mu \overleftrightarrow{D}_\mu R_A,$$

$$\mathcal{L}_{quark} = \frac{1}{2} i \bar{Q}_A \gamma^\mu \overleftrightarrow{D}_\mu Q_A + \frac{1}{2} i \bar{U}_A \gamma^\mu \overleftrightarrow{D}_\mu U_A + \frac{1}{2} i \bar{D}_A \gamma^\mu \overleftrightarrow{D}_\mu D_A,$$

$$\mathcal{L}_{Yukawa} = -[(G_L)_{AB} \bar{L}_A \phi R_B + (G_U)_{AB} \bar{L}_A \phi^c R_B + (G_D)_{AB} \bar{Q}_A \phi D_B] + H.C.$$

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2,$$

$$\mathcal{L}_{Gauge} = -\frac{1}{2} Tr(G_{\mu\nu} G^{\mu\nu}) - \frac{1}{2} Tr(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

Standard Model of Particles + LIV SSB

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$$\mathcal{L}_{Gauge} = -\frac{1}{2} Tr(G_{\mu\nu} G^{\mu\nu}) - \frac{1}{2} Tr(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \dots$$

The (Minimal) Standard Model Extension

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} = \frac{1}{2} i(c_L)_{\mu\nu AB} \bar{L}_A \gamma^\mu \overset{\leftrightarrow}{D}^\nu L_B + \frac{1}{2} i(c_R)_{\mu\nu AB} \bar{R}_A \gamma^\mu \overset{\leftrightarrow}{D}^\nu R_B$$

$$\mathcal{L}_{\text{gauge}}^{\text{CPT-even}} = -\frac{1}{2} (k_G)_{\kappa\lambda\mu\nu} \text{Tr}(G^{\kappa\lambda} G^{\mu\nu})$$

$$-\frac{1}{2} (k_W)_{\kappa\lambda\mu\nu} \text{Tr}(W^{\kappa\lambda} W^{\mu\nu}) - \frac{1}{4} (k_B)_{\kappa\lambda\mu\nu} B^{\kappa\lambda} B^{\mu\nu}$$

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} = -\frac{1}{2} i(a_L)_{\mu AB} \bar{L}_A \gamma^\mu L_B - \frac{1}{2} i(a_R)_{\mu AB} \bar{R}_A \gamma^\mu R_B$$

$$+ \mathcal{L}_{\text{CPT-even}}^{\text{quark}} + \mathcal{L}_{\text{CPT-odd}}^{\text{quark}} + \mathcal{L}_{\text{CPT-even}}^{\text{Yukawa}} + \mathcal{L}_{\text{CPT-even}}^{\text{Higgs}} + \mathcal{L}_{\text{CPT-odd}}^{\text{Higgs}} + \mathcal{L}_{\text{CPT-odd}}^{\text{gauge}}$$

Properties conserved

Energy-momentum conservation, hermiticity, micro-causality, positivity of the energy, power counting renormalizable, O. Lorentz invariance, ...

$$\begin{aligned}\mathcal{L}_{LIVQED} = & i\bar{\psi}\gamma^\mu D_\mu\psi - m^2\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + ix\psi\gamma^i D_i\psi + \frac{ig}{M^2}D^j\bar{\psi}\gamma^i D_i D_j\psi + \frac{\epsilon}{4M^2}F_{kj}\partial_i^2 F^{kj}.\end{aligned}$$

Where $D_\mu\psi = (\partial_\mu + ieA_\mu)\psi$

- Gauge invariance preserved.
- Invariant under rotations in three-dimensional space in the preferred frame.
- CPT and P invariance.
- Only operators that cannot be removed by a field and/or coordinate redefinition