technische universität
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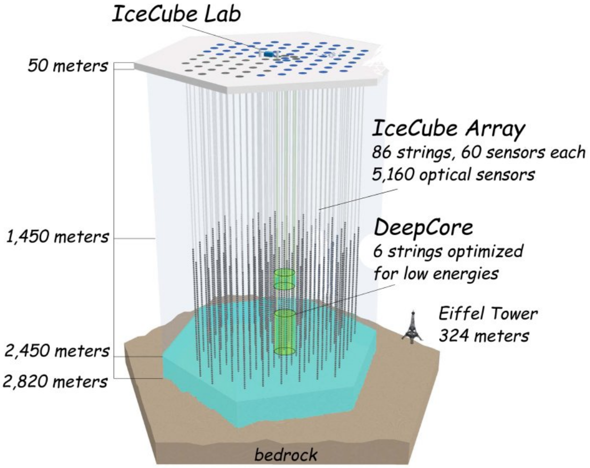
Higher order corrections to muon cross sections

Astroparticle School, October 7—15 2015

Alexander Sandrock

The IceCube Neutrino Observatory



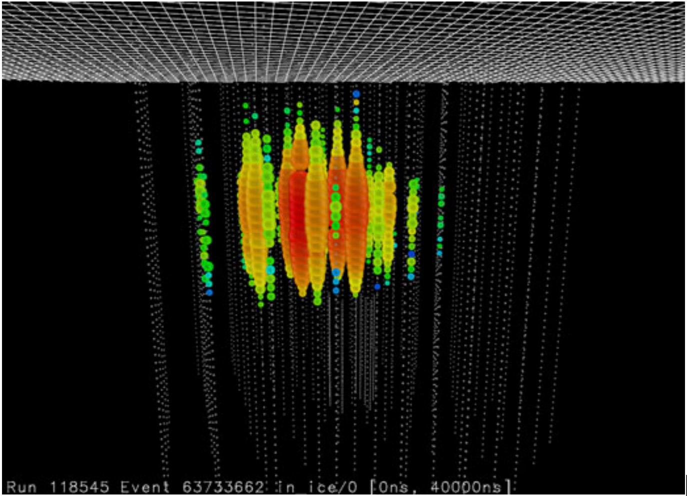
[1]

Neutrino events in IceCube

Neutrino events in IceCube can be broadly categorized in cascade-like events and track-like events.

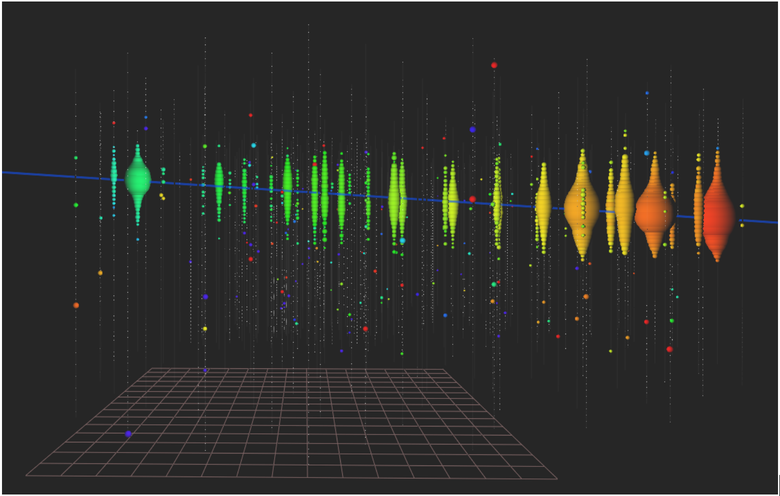
- Cascade-like events are produced by weak neutral-current interactions of neutrinos of all flavours, and by weak charged-current interactions of electrons and tau leptons.
- Track-like events are produced by charged-current interactions of muon neutrinos (and also by atmospheric muons).

Neutrino events in IceCube



[2]

Neutrino events in IceCube



[3]

Energy reconstruction

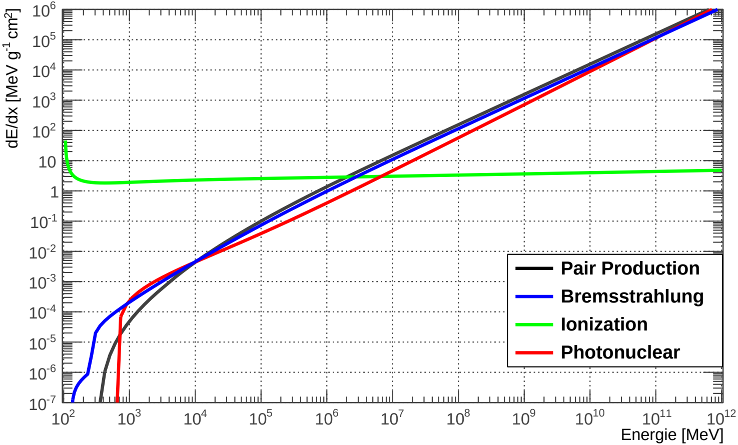
- For cascade-like events IceCube can be viewed as a calorimeter.
- The energy of starting tracks can be reconstructed from the hadronic cascade of the primary interaction.
- The energy of through-going tracks can only be estimated based on the energy losses dE/dx

$$-\frac{dE}{dx} = a + b(E)E, \quad b(E) = \frac{N_0}{A} \int v\sigma(E, v)\, dv$$

$\Rightarrow$ The energy losses must be known very well for a good energy reconstruction.

Energy losses

Charged particles loose energy in the detector by ionization, pair



[4]

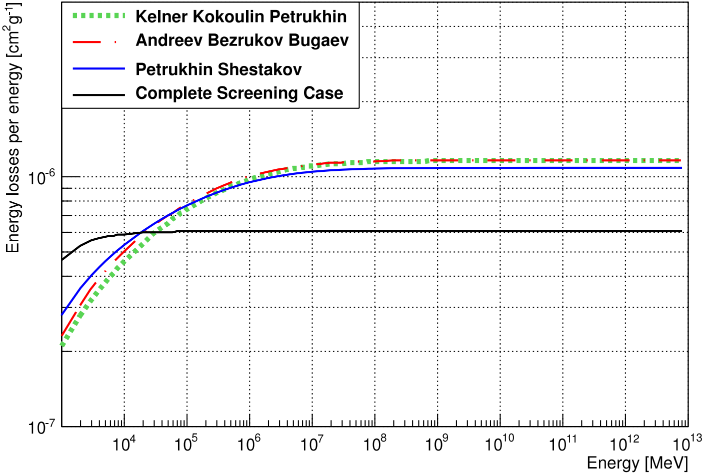
Parametrizations of bremsstrahlung

Currently IceCube uses three parametrizations of the muon bremsstrahlung cross section:

- Kelner-Kokoulin-Petrukhin (1995)
- Petrukhin-Shestakov (1966)
- Andreev-Bezrukov-Bugaev (1994)

plus corrections for bremsstrahlung on atomic electrons and target excitation in bremsstrahlung.

Energy losses for the different parametrizations



[4]

Higher order calculation

For QED on atoms there are two perturbative expansions to be taken into account:

- the expansion for the radiation field $\mathcal{O}(\alpha)$ (radiative corrections),
- and the expansion for the coupling to the nuclear field $\mathcal{O}(Z\alpha)$ (Coulomb corrections).

Radiative corrections

- Radiative corrections can in principle be calculated using the normal tools of quantum field theory.
- Special care has to be taken, because numerical problems make the phase-space integration complicated.

Coulomb corrections

- Coulomb corrections require a different approach, because for heavy nuclei $Z\alpha$ is not small (non-perturbative QED).
- Possible ansatz is the use of quasiclassical wave-functions, which are small-angle approximations to the exact solution of the Dirac equation in the potential of the nucleus.

Some preliminary results for radiative corrections

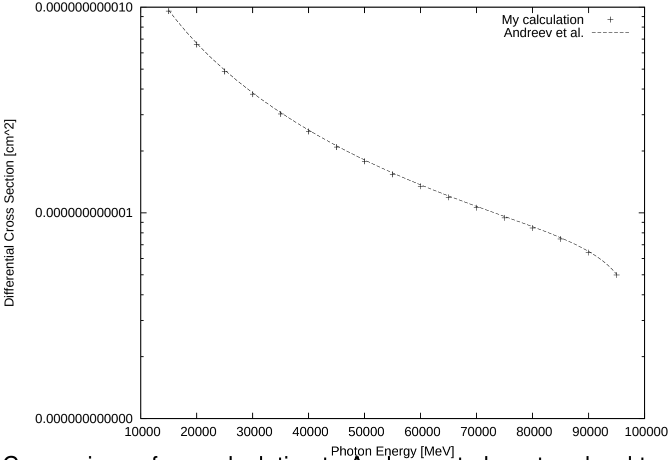


Figure: Comparison of my calculation to Andreev et al. on tree level to prove correctness for initial muon energy of 100 GeV.

Some preliminary results

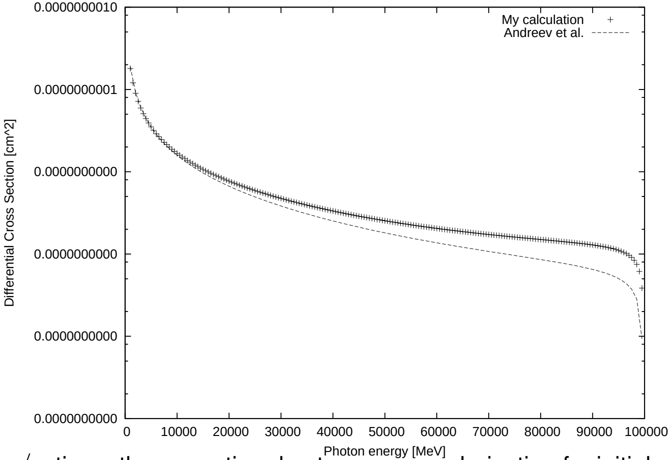


Figure: π/α times the correction due to vacuum polarization for initial muon energy of 100 GeV.

Landau-Pomeranchuk-Migdal and Ter-Mikaelyan effects

- The previous calculations consider only the interaction with an isolated atom.
- If in the coherence length there is only one atom that is OK.
- The coherence length $\ell_C = \hbar/q$ with q the momentum transfer can reach macroscopic dimensions for very high energies.
- For big coherence length one has to take into account the disturbance of the wave functions by multiple scattering of the muon (LPM effect) and Compton scattering of the photon (TM effect).
- If scattering occurs inside the emission region, bremsstrahlung is suppressed.

Conclusion

- Energy reconstruction of neutrino events is based on the energy losses.
- The energy losses depend on the cross sections.

⇒ Accurate knowledge of energy loss cross sections is crucial for neutrino telescopes.

Image sources

- *Spencer Klein:* Into the Ice: Completing the IceCube Neutrino Observatory. `http://newscenter.lbl.gov/2010/12/17/completing-icecube/`
- *Laurel Norris:* IceCube reveals interesting high-energy neutrino events. `http://icecube.wisc.edu/news/view/105`
- *Silvia Bravo:* Designing the future of the IceCube Neutrino Observatory. `http://icecube.wisc.edu/news/view/286`
- *Jan Hendrik Koehne:* Der Leptonpropagator PROPOSAL. PhD Thesis, Technische Universität Dortmund, 2013.

Backup slides

Passarino-Veltman decomposition of tensor integrals

Tensor integrals of the form

$$T_n^{\mu_1 \cdots \mu_\ell} = \int \frac{d^n q}{(2\pi)^n} \frac{q^{\mu_1} \cdots q^{\mu_\ell}}{\prod_{i=0}^{m} [(q+p_i)^2 - m_i^2]}$$

can be decomposed in coefficient functions and tensor coefficients built from the metrical tensor $g^{\mu\nu}$ and the external vectors p_i, e.g.

$$\begin{aligned} B^{\mu\nu} &= \int \frac{d^n q}{(2\pi)^n} \frac{q^\mu q^\nu}{(q^2 - m_0^2)[(q+p_1)^2 - m_1^2]} \\ &= g^{\mu\nu} B_{00}(p_1^2, m_0^2, m_1^2) + p_1^\mu p_1^\nu B_{11}(p_1^2, m_0^2, m_1^2). \end{aligned}$$

Padé approximants

Padé approximants are a generalization of Taylor series and have the form

$$f(x) = \frac{\sum_{i=0}^{m} a_i x^i}{1 + \sum_{k=1}^{n} b_k x^k}.$$

Padé approximants can sometimes even exist at points where Taylor series are undefined or converge very slowly.